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Exercise 7. Stab(x) is a subgroup of G

G a group, $x \in X$ a G-set and

$$\mathrm{Stab}(x) = \{ g \in G \mid g \cdot x = x \}$$

To be verified: Stab(x) satisfies the group axioms w.r.t. $\cdot$

(associative) (i) Since for every $g \in G$ is associative as G is a group and $\mathrm{Stab}(x) = \{ g \in G \mid g \cdot x = x \}$ it follows that $\forall g' \in \mathrm{Stab}(x)$ is associative

(ii) Let $e_G \in G$ identity, $e_G \cdot x \overset{def}{=} x \Rightarrow e_G \in \mathrm{Stab}(x)$
(identity)

(closure) (iii) Let $a, b \in \mathrm{Stab}(x)$, then $(a \cdot b) \cdot x$

$$= a \cdot (b \cdot x) = a \cdot x = x \Rightarrow (a \cdot b) \in \mathrm{Stab}(x)$$

(inverses) (iv) Let $a \in \mathrm{Stab}(x)$, $a \cdot x = x$

then $\exists a^{-1} \in G$ and then $a^{-1} \cdot x = a^{-1} \cdot (a \cdot x)$

$$\overset{assoc.}{=} (a^{-1} a) x = e_G \cdot x = x \Rightarrow a^{-1} \in \mathrm{Stab}(x)$$

$\therefore$ Stab(x) subgroup of G

$\blacksquare$