

Ex 8. Suppose $\text{ORBIT}(x)$ and $\text{ORBIT}(y)$ have a common element z :

$$z = g_1 x \quad , \quad z = g_2 y.$$

We want to show $\text{ORBIT}(x) = \text{ORBIT}(y)$ For any point $u \in \text{ORBIT}(x)$, we write $u = g \cdot x$ for some $g \in G$. Since $x = g_1^{-1} z$

$$u = g(g_1^{-1} z) = (gg_1^{-1})z = (gg_1^{-1})(g_2 y) = (gg_1^{-1}g_2) \cdot y.$$

which show $u \in \text{ORBIT}(y)$ Therefore $\text{ORBIT}(x) \subset \text{ORBIT}(y)$. Similarly we can prove $\text{ORBIT}(y) \subset \text{ORBIT}(x)$ Thus we proved.

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