PRACTICE FOR MIDTERM 1

MIDTERM TO TAKE PLACE MARCH 28, 2014

The exam on Wednesday will be a learning exam. You will be asked to solve problems on the board with the help of students in the class. Think about what we did in this class so far: we have learned to take finite groups and their representations (modules) and break them down into irreducible representations (irreducible modules). The midterm will be a check that we are in the right place in this material. That is, that I can give each of you a finite group (probably of size ≤ 12) and lead you through the steps of creating its character table, then given some module of that group ask you to tell me how it breaks down into irreducible components.

Let D_4 be the group generated by elements x, y with $x^4 = y^2 = e$ and $xy = yx^{-1}$.

(1) Show that the conjugacy classes of D_4 are
$$\{e\} \cup \{x, x^3\} \cup \{x^2\} \cup \{y, yx^2\} \cup \{yx, yx^3\} \ .$$
(2) Find the 1-dimensional representations/characters of D_4.
(3) Find the number and dimensions of the non-1-dimensional representations of D_4.
(4) Find the character table of D_4.

Now consider the D_4 module $\mathcal{L}\{a_0, a_1, b_0, b_1\}$ with the action $xa_i = a_{1-i}$ and $ya_i = b_i$.
(5) Prove that this does in fact define a module.
(6) Find the action of the group elements on the basis and compute the character of this module.
(7) Find the multiplicities of each of the irreducible modules in this module by computing the scalar products of the character of this module with the character table.
(8) Find the decomposition of this module into irreducible components (find a decomposition into irreducible submodules).

Now consider the space of polynomials in the variables a_0, a_1, b_0, b_1. Let $P = \mathbb{C}[a_0, a_1, b_0, b_1]$ and let P_k be the polynomials of degree k so that $P = \bigoplus_{k \geq 0} P_k$.
(9) What is the dimension of the polynomials of degree k?
(10) Use the group action to state the action of the generators of D_4 on the basis elements of the form $a_0^i a_1^j b_0^k b_1^\ell$ of P so that P_k is a submodule for any $k \geq 0$.
(11) For $k = 2$ find the character of this module
(12) For $k = 2$ find the multiplicities of each of the irreducibles.