

1. CONJECTURED TABLEAUX RANK FUNCTIONS FOR  $n = 5$

1.1.  $G_\lambda[X; q]$  for  $\lambda$  strict partition of 5.

$$\begin{aligned} G_{3,2}^q &= 2q^2Q_5 + 2qQ_{4,1} + Q_{3,2} \\ G_{4,1}^q &= 2qQ_5 + Q_{4,1} \\ G_5^q &= Q_5 \end{aligned}$$

1.2. Conjectured tableaux rank function of content  $(3, 2)$ .

$$G_{3,2}^q = 2q^2Q_5 + 2qQ_{4,1} + Q_{3,2}$$

	2	2	
1	1	1	
	2		
1	1	1	2*
1	1	1	2*
	2		

1.3. Conjectured tableaux rank function of content  $(4, 1)$ .

$$G_{4,1}^q = 2qQ_5 + Q_{4,1}$$

	2			
1	1	1	1	
1	1	1	1	2*

## 2. CONJECTURED TABLEAUX RANK FUNCTIONS FOR $n = 6$

### 2.1. $G_\lambda[X; q]$ for $\lambda$ strict partition of 6.

$$\begin{aligned} G_{3,2,1}^q &= 4q^4Q_6 + (4q^3 + 2q^2)Q_{5,1} + (4q^2 + 2q)Q_{4,2} + Q_{3,2,1} \\ G_{4,2}^q &= 2q^2Q_6 + 2qQ_{5,1} + Q_{4,2} \\ G_{5,1}^q &= 2qQ_6 + Q_{5,1} \\ G_6^q &= Q_6 \end{aligned}$$

### 2.2. Conjectured tableaux rank function of content $(3, 2, 1)$ .

$$4q^4Q_6 + (2q^2 + 4q^3)Q_{5,1} + (2q + 4q^2)Q_{4,2} + Q_{3,2,1}$$

	<table border="1"><tr><td>3</td></tr><tr><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	3	2	2	1	1	1	
3								
2	2							
1	1	1						
	<table border="1"><tr><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td><td>3*</td></tr></table>	2	2	1	1	1	3*	
2	2							
1	1	1	3*					
	<table border="1"><tr><td>2</td><td>3*</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2*</td></tr></table>	2	3*	1	1	1	2*	
2	3*							
1	1	1	2*					
	<table border="1"><tr><td>3</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2*</td><td>2</td></tr></table>	3	1	1	1	2*	2	
3								
1	1	1	2*	2				
	<table border="1"><tr><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>2*</td><td>3*</td></tr></table>	2	1	1	1	1	2*	3*
2								
1	1	1	1	2*	3*			
	<table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>2*</td><td>2</td><td>3*</td></tr></table>	1	1	1	1	2*	2	3*
1	1	1	1	2*	2	3*		

### 2.3. Conjectured tableaux rank function of content $(4, 2)$ .

$$G_{4,2}^q = 2q^2Q_6 + 2qQ_{5,1} + Q_{4,2}$$

	<table border="1"><tr><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table>	2	2	1	1	1	1
2	2						
1	1	1	1				
	<table border="1"><tr><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>2*</td></tr></table>	2	1	1	1	1	2*
2							
1	1	1	1	2*			
	<table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>2*</td><td>2</td></tr></table>	1	1	1	1	2*	2
1	1	1	1	2*	2		

### 2.4. Conjectured tableaux rank function of content $(5, 1)$ .

$$G_{5,1}^q = 2qQ_6 + Q_{5,1}$$

	<table border="1"><tr><td>2</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr></table>	2	1	1	1	1	1
2							
1	1	1	1	1			
	<table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>2*</td></tr></table>	1	1	1	1	1	2*
1	1	1	1	1	2*		

### 3. CONJECTURED TABLEAUX RANK FUNCTIONS FOR $n = 7$

#### 3.1. $G_\lambda[X; q]$ for $\lambda$ strict partition of 7.

$$\begin{aligned} G_{4,2,1}^q &= 4q^4Q_7 + (4q^3 + 2q^2)Q_{6,1} + (4q^2 + 2q)Q_{5,2} + 2qQ_{4,3} + Q_{4,2,1} \\ G_{4,3}^q &= 2q^3Q_7 + 2q^2Q_{6,1} + 2qQ_{5,2} + Q_{4,3} \\ G_{5,2}^q &= 2q^2Q_7 + 2qQ_{6,1} + Q_{5,2} \\ G_{6,1}^q &= 2qQ_7 + Q_{6,1} \\ G_7^q &= Q_7 \end{aligned}$$

#### 3.2. Conjectured tableaux rank function of content $(4, 2, 1)$ .

$$G_{4,2,1}^q = 4q^4Q_7 + (4q^3 + 2q^2)Q_{6,1} + (2q + 4q^2)Q_{5,2} + 2qQ_{4,3} + Q_{4,2,1}$$

$\begin{array}{ c c c c } \hline & & 3 \\ \hline & 2 & 2 \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array}$
$\begin{array}{ c c c c } \hline 2 & 2 & 3^* \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array}$
$\begin{array}{ c c c c } \hline 2 & 2 \\ \hline 1 & 1 & 1 & 1 \\ \hline 3^* \\ \hline \end{array}$
$\begin{array}{ c c c c c } \hline 2 & 3^* \\ \hline 1 & 1 & 1 & 1 & 2^* \\ \hline \end{array}$

$\begin{array}{ c c c c c } \hline 2 \\ \hline 1 & 1 & 1 & 1 & 2^* \\ \hline 3^* \\ \hline \end{array}$
$\begin{array}{ c c c c c c } \hline 1 & 1 & 1 & 1 & 2^* & 2 & 3^* \\ \hline \end{array}$

#### 3.3. Conjectured tableaux rank function of content $(4, 3)$ .

$$G_{4,3}^q = 2q^3Q_7 + 2q^2Q_{6,1} + 2qQ_{5,2} + Q_{4,3}$$

$\begin{array}{ c c c c c } \hline 2 & 2 & 2 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array}$
$\begin{array}{ c c c c c } \hline 2 & 2 \\ \hline 1 & 1 & 1 & 1 & 2^* \\ \hline \end{array}$
$\begin{array}{ c c c c c c } \hline 2 \\ \hline 1 & 1 & 1 & 1 & 2^* & 2 \\ \hline \end{array}$
$\begin{array}{ c c c c c c c } \hline 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \\ \hline \end{array}$

#### 3.4. Conjectured tableaux rank function of content $(5, 2)$ .

$$G_{5,2}^q = 2q^2Q_7 + 2qQ_{6,1} + Q_{5,2}$$

$\begin{array}{ c c c c c c } \hline 2 & 2 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ \hline \end{array}$
$\begin{array}{ c c c c c c c } \hline 2 \\ \hline 1 & 1 & 1 & 1 & 1 & 2^* \\ \hline \end{array}$
$\begin{array}{ c c c c c c c } \hline 1 & 1 & 1 & 1 & 1 & 2^* & 2 \\ \hline \end{array}$

**3.5. Conjectured tableaux rank function of content  $(6, 1)$ .**

$$G_{6,1}^q = 2qQ_7 + Q_{6,1}$$

2
1 1 1 1 1 1 1
1 1 1 1 1 1 2*

#### 4. CONJECTURED TABLEAUX RANK FUNCTIONS FOR $n = 8$

##### 4.1. $G_\lambda[X; q]$ for $\lambda$ strict partition of 8.

$$\begin{aligned}
 G_{4,3,1}^q &= 4q^5Q_8 + (2q^3 + 4q^4)Q_{7,1} + (4q^3 + 4q^2)Q_{6,2} + (4q^2 + 2q)Q_{5,3} + 2qQ_{5,2,1} + Q_{4,3,1} \\
 G_{5,2,1}^q &= 4q^4Q_8 + (2q^2 + 4q^3)Q_{7,1} + (4q^2 + 2q)Q_{6,2} + 2qQ_{5,3} + Q_{5,2,1} \\
 G_{5,3}^q &= 2q^3Q_8 + 2q^2Q_{7,1} + 2qQ_{6,2} + Q_{5,3} \\
 G_{6,2}^q &= 2q^2Q_8 + 2qQ_{7,1} + Q_{6,2} \\
 G_{7,1}^q &= 2qQ_8 + Q_{7,1} \\
 G_8^q &= Q_8
 \end{aligned}$$

##### 4.2. Conjectured tableaux rank function of content $(4, 3, 1)$ .

$$G_{4,3,1}^q = 4q^5Q_8 + (2q^3 + 4q^4)Q_{7,1} + (4q^2 + 4q^3)Q_{6,2} + (2q + 4q^2)Q_{5,3} + 2qQ_{5,2,1} + Q_{4,3,1}$$

<p><math>\begin{array}{c} 3 \\ 2\ 2\ 2 \\ 1\ 1\ 1\ 1 \end{array}</math></p>
<p><math>\begin{array}{c} 3 \\ 2\ 2 \\ 1\ 1\ 1\ 1\ 2^* \end{array}</math>    <math>\begin{array}{c} 2\ 2\ 2 \\ 1\ 1\ 1\ 1\ 3^* \end{array}</math></p>
<p><math>\begin{array}{c} 2\ 2\ 3^* \\ 1\ 1\ 1\ 1\ 2^* \end{array}</math>    <math>\begin{array}{c} 2\ 2 \\ 1\ 1\ 1\ 1\ 2^* 3^* \end{array}</math></p>
<p><math>\begin{array}{c} 2\ 3^* \\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}</math>    <math>\begin{array}{c} 3 \\ 1\ 1\ 1\ 1\ 2^* 2\ 2 \end{array}</math></p>
<p><math>\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 2^* 2\ 3^* \end{array}</math></p>
<p><math>\begin{array}{c} 1\ 1\ 1\ 1\ 2^* 2\ 2\ 3^* \end{array}</math></p>

##### 4.3. Conjectured tableaux rank function of content $(5, 2, 1)$ .

$$G_{5,2,1}^q = 4q^4Q_8 + (2q^2 + 4q^3)Q_{7,1} + (4q^2 + 2q)Q_{6,2} + 2qQ_{5,3} + Q_{5,2,1}$$

<p><math>\begin{array}{c} 3 \\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1 \end{array}</math></p>
<p><math>\begin{array}{c} 2\ 2\ 3^* \\ 1\ 1\ 1\ 1\ 1 \end{array}</math>    <math>\begin{array}{c} 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 3^* \end{array}</math></p>
<p><math>\begin{array}{c} 2\ 3^* \\ 1\ 1\ 1\ 1\ 1\ 2^* \end{array}</math>    <math>\begin{array}{c} 3 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}</math></p>
<p><math>\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 3^* \end{array}</math></p>
<p><math>\begin{array}{c} 1\ 1\ 1\ 1\ 1\ 2^* 2\ 3^* \end{array}</math></p>

#### 4.4. Conjectured tableaux rank function of content $(5, 3)$ .

$$G_{5,3}^q = 2 q^3 Q_8 + 2 q^2 Q_{7,1} + 2 q Q_{6,2} + Q_{5,3}$$

	2	2	2
1	1	1	1
	2	2	
1	1	1	1
1	1	1	2*
	2		
1	1	1	1
1	2*	2	
1	1	1	1
1	1	2*	2
1	1	1	2

#### 4.5. Conjectured tableaux rank function of content $(6, 2)$ .

$$G_{6,2}^q = 2q^2Q_8 + 2qQ_{7,1} + Q_{6,2}$$

	2	2	
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	2*
1	1	1	1
1	1	1	2*

#### 4.6. Conjectured tableaux rank function of content $(7, 1)$ .

$$G_{7,1}^q = 2qQ_8 + Q_{7,1}$$

	2						
1	1	1	1	1	1	1	
1	1	1	1	1	1	1	2*

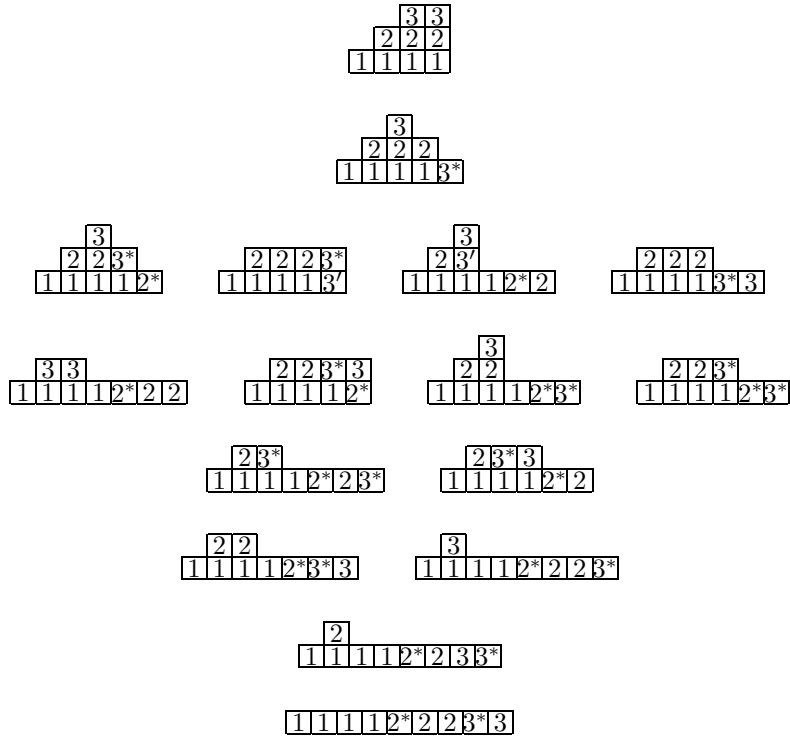
## 5. CONJECTURED RANK FUNCTIONS FOR $n = 9$

### 5.1. $G_\lambda[X; q]$ for $\lambda$ strict partition of 9.

$$\begin{aligned}
G_{4,3,2}^q &= 4q^7Q_9 + (4q^5 + 4q^6)Q_{8,1} + (2q^3 + 8q^4 + 4q^5)Q_{7,2} + (8q^3 + 4q^4 + 2q^2)Q_{6,3} \\
&\quad + (4q^3 + 2q^2)Q_{6,2,1} + (4q^3 + 2q^2)Q_{5,4} + (4q^2 + 2q)Q_{5,3,1} + Q_{4,3,2} \\
G_{5,3,1}^q &= 4q^5Q_9 + (4q^4 + 2q^3)Q_{8,1} + (4q^3 + 4q^2)Q_{7,2} + (4q^2 + 2q)Q_{6,3} \\
&\quad + 2qQ_{6,2,1} + 2qQ_{5,4} + Q_{5,3,1} \\
G_{5,4}^q &= 2q^4Q_9 + 2q^3Q_{8,1} + 2q^2Q_{7,2} + 2qQ_{6,3} + Q_{5,4} \\
G_{6,2,1}^q &= 4q^4Q_9 + (4q^3 + 2q^2)Q_{8,1} + (4q^2 + 2q)Q_{7,2} + 2qQ_{6,3} + Q_{6,2,1} \\
G_{6,3}^q &= 2q^3Q_9 + 2q^2Q_{8,1} + 2qQ_{7,2} + Q_{6,3} \\
G_{7,2}^q &= 2q^2Q_9 + 2qQ_{8,1} + Q_{7,2} \\
G_{8,1}^q &= 2qQ_9 + Q_{8,1} \\
G_9^q &= Q_9
\end{aligned}$$

## 5.2. Conjectured tableaux rank function of content $(4, 3, 2)$ .

$$G_{4,3,2}^q = 4 q^7 Q_9 + (4 q^5 + 4 q^6) Q_{8,1} + (2 q^3 + 4 q^5 + 8 q^4) Q_{7,2} + (2 q^2 + 8 q^3 + 4 q^4) Q_{6,3} \\ + (4 q^3 + 2 q^2) Q_{6,2,1} + (4 q^3 + 2 q^2) Q_{5,4} + (4 q^2 + 2 q) Q_{5,3,1} + Q_{4,3,2}$$



**5.3. Conjectured tableaux rank function of content (5, 3, 1).**

$$\begin{aligned} G_{5,3,1}^q = & 4 q^5 Q_9 + (2 q^3 + 4 q^4) Q_{8,1} + (4 q^3 + 4 q^2) Q_{7,2} + (4 q^2 + 2 q) Q_{6,3} \\ & + 2 q Q_{6,2,1} + 2 q Q_{5,4} + Q_{5,3,1} \end{aligned}$$

 $\begin{array}{c} 3 \\ 2\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1 \end{array}$	 $\begin{array}{c} 3 \\ 2\ 2 \\ 1\ 1\ 1\ 1\ 2^* \end{array}$	 $\begin{array}{c} 2\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 3^* \end{array}$
 $\begin{array}{c} 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 3^* \end{array}$	 $\begin{array}{c} 2\ 2\ 3^* \\ 1\ 1\ 1\ 1\ 1\ 2^* \end{array}$	 $\begin{array}{c} 2\ 3^* \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$
 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$	 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$	 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$

**5.4. Conjectured tableaux rank function of content (6, 2, 1).**

$$G_{6,2,1}^q = 4 q^4 Q_9 + (2 q^2 + 4 q^3) Q_{8,1} + (4 q^2 + 2 q) Q_{7,2} + 2 q Q_{6,3} + Q_{6,2,1}$$

 $\begin{array}{c} 3 \\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 1 \end{array}$	 $\begin{array}{c} 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 3^* \end{array}$	 $\begin{array}{c} 2\ 2\ 3^* \\ 1\ 1\ 1\ 1\ 1\ 1 \end{array}$
 $\begin{array}{c} 2\ 3^* \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* \end{array}$	 $\begin{array}{c} 3 \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$	 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* 3^* \end{array}$
 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$	 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$	 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$

**5.5. Conjectured tableaux rank function of content (5, 4).**

$$G_{5,4}^q = 2 q^4 Q_9 + 2 q^3 Q_{8,1} + 2 q^2 Q_{7,2} + 2 q Q_{6,3} + Q_{5,4}$$

 $\begin{array}{c} 2\ 2\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1 \end{array}$
 $\begin{array}{c} 2\ 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* \end{array}$
 $\begin{array}{c} 2\ 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$
 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$
 $\begin{array}{c} 2 \\ 1\ 1\ 1\ 1\ 1\ 2^* 2 \end{array}$

### 5.6. Conjectured tableaux rank function of content $(6, 3)$ .

$$G_{6,3}^q = 2 q^3 Q_9 + 2 q^2 Q_{8,1} + 2 q Q_{7,2} + Q_{6,3}$$

	2	2	2	
1	1	1	1	1
	2	2		
1	1	1	1	1
1	1	1	1	1

### 5.7. Conjectured tableaux rank function of content $(7, 2)$ .

$$G_{7,2}^q = 2q^2Q_9 + 2qQ_{8,1} + Q_{7,2}$$

### 5.8. Conjectured tableaux rank function of content $(8, 1)$ .

$$G_{8,1}^q = 2qQ_9 + Q_{8,1}$$

## 6. CONJECTURED RANK FUNCTIONS FOR $n = 10$

### 6.1. $G_\lambda[X; q]$ for $\lambda$ strict partition of 10.

$$\begin{aligned}
G_{4,3,2,1}^q &= 8q^{10}Q_{10} + (4q^7 + 8q^9 + 8q^8)Q_{9,1} + (4q^5 + 16q^7 + 16q^6 + 8q^8)Q_{8,2} \\
&\quad + (8q^4 + 24q^5 + 16q^6 + 8q^7)Q_{7,3} + (2q^3 + 4q^4 + 12q^5 + 8q^6)Q_{7,2,1} \\
&\quad + (4q^3 + 12q^4 + 8q^6 + 16q^5)Q_{6,4} + (2q^2 + 8q^3 + 20q^4 + 8q^5)Q_{6,3,1} \\
&\quad + (2q^2 + 8q^3 + 8q^4)Q_{5,4,1} + (2q + 4q^2 + 8q^3)Q_{5,3,2} + Q_{4,3,2,1} \\
G_{5,3,2}^q &= 4q^7Q_{10} + (4q^5 + 4q^6)Q_{9,1} + (4q^5 + 2q^3 + 8q^4)Q_{8,2} + (4q^4 + 2q^2 + 8q^3)Q_{7,3} \\
&\quad + (2q^2 + 4q^3)Q_{7,2,1} + (4q^2 + 4q^3)Q_{6,4} + (2q + 4q^2)Q_{6,3,1} + 2qQ_{5,4,1} + Q_{5,3,2} \\
G_{5,4,1}^q &= 4q^6Q_{10} + (2q^4 + 4q^5)Q_{9,1} + (4q^4 + 4q^3)Q_{8,2} + (4q^2 + 4q^3)Q_{7,3} \\
&\quad + 2q^2Q_{7,2,1} + (2q + 4q^2)Q_{6,4} + 2qQ_{6,3,1} + Q_{5,4,1} \\
G_{6,3,1}^q &= 4q^5Q_{10} + (2q^3 + 4q^4)Q_{9,1} + (4q^2 + 4q^3)Q_{8,2} + (2q + 4q^2)Q_{7,3} + 2qQ_{7,2,1} \\
&\quad + 2qQ_{6,4} + Q_{6,3,1} \\
G_{6,4}^q &= 2q^4Q_{10} + 2q^3Q_{9,1} + 2q^2Q_{8,2} + 2qQ_{7,3} + Q_{6,4} \\
G_{7,2,1}^q &= 4q^4Q_{10} + (4q^3 + 2q^2)Q_{9,1} + (4q^2 + 2q)Q_{8,2} + 2qQ_{7,3} + Q_{7,2,1} \\
G_{7,3}^q &= 2q^3Q_{10} + 2q^2Q_{9,1} + 2qQ_{8,2} + Q_{7,3} \\
G_{8,2}^q &= 2q^2Q_{10} + 2qQ_{9,1} + Q_{8,2} \\
G_{9,1}^q &= 2qQ_{10} + Q_{9,1} \\
G_{10}^q &= Q_{10}
\end{aligned}$$

### 6.2. Conjectured tableaux rank function of content $(4, 3, 2, 1)$ .

$$\begin{aligned}
G_{4,3,2,1}^q &= 8q^{10}Q_{10} + (4q^7 + 8q^9 + 8q^8)Q_{9,1} + (4q^5 + 16q^7 + 16q^6 + 8q^8)Q_{8,2} \\
&\quad + (8q^4 + 24q^5 + 16q^6 + 8q^7)Q_{7,3} + (2q^3 + 4q^4 + 12q^5 + 8q^6)Q_{7,2,1} \\
&\quad + (4q^3 + 12q^4 + 8q^6 + 16q^5)Q_{6,4} + (2q^2 + 8q^3 + 20q^4 + 8q^5)Q_{6,3,1} \\
&\quad + (2q^2 + 8q^3 + 8q^4)Q_{5,4,1} + (2q + 4q^2 + 8q^3)Q_{5,3,2} + Q_{4,3,2,1}
\end{aligned}$$

### 6.3. Conjectured tableaux rank function of content $(5, 3, 2)$ .

$$\begin{aligned} G_{5,3,2}^q = & 4q^7 Q_{10} + (4q^5 + 4q^6) Q_{9,1} + (4q^5 + 2q^3 + 8q^4) Q_{8,2} + (4q^4 + 2q^2 + 8q^3) Q_{7,3} \\ & + (2q^2 + 4q^3) Q_{7,2,1} + (4q^2 + 4q^3) Q_{6,4} + (2q + 4q^2) Q_{6,3,1} + 2qQ_{5,4,1} + Q_{5,3,2} \end{aligned}$$

			$\begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$
		$\begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 3^* \end{array}$
$\begin{array}{c} 3 \\ 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 3^* \end{array}$	$\begin{array}{c} 3 \\ 2 \\ 3' \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 3^* \\ 3 \end{array}$
$\begin{array}{c} 3 \\ 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 3^* \\ 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 3 \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 3^* \end{array}$
	$\begin{array}{c} 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 3 \end{array}$	$\begin{array}{c} 2 \\ 3^* \\ 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \end{array}$	
$\begin{array}{c} 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 3^* \\ 3 \end{array}$	$\begin{array}{c} 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \\ 3^* \end{array}$		
		$\begin{array}{c} 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 3 \\ 3^* \end{array}$	
			$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \\ 3^* \\ 3 \end{array}$

### 6.4. Conjectured tableaux rank function of content $(5, 4, 1)$ .

$$\begin{aligned} G_{5,4,1}^q = & 4q^6 Q_{10} + (2q^4 + 4q^5) Q_{9,1} + (4q^4 + 4q^3) Q_{8,2} + (4q^2 + 4q^3) Q_{7,3} \\ & + 2q^2 Q_{7,2,1} + (2q + 4q^2) Q_{6,4} + 2qQ_{6,3,1} + Q_{5,4,1} \end{aligned}$$

			$\begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$
		$\begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 3^* \end{array}$
$\begin{array}{c} 2 \\ 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 3 \end{array}$	
$\begin{array}{c} 2 \\ 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \end{array}$	$\begin{array}{c} 2 \\ 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \end{array}$		
$\begin{array}{c} 2 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 3^* \end{array}$	$\begin{array}{c} 3^* \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \\ 2 \end{array}$		
		$\begin{array}{c} 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \\ 3^* \end{array}$	
			$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2^* \\ 2 \\ 2 \\ 2 \\ 3^* \end{array}$

**6.5. Conjectured tableaux rank function of content (6, 3, 1).**

$$G_{6,3,1}^q = 4q^5Q_{10} + (2q^3 + 4q^4)Q_{9,1} + (4q^2 + 4q^3)Q_{8,2} + (2q + 4q^2)Q_{7,3} + 2qQ_{7,2,1} + 2qQ_{6,4} + Q_{6,3,1}$$

 $\begin{matrix} & & 3 \\ & & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$
 $\begin{matrix} & & 3 \\ & & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* \end{matrix}$
 $\begin{matrix} & & 3 \\ & & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3^* \end{matrix}$
 $\begin{matrix} & & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 3^* \end{matrix}$
 $\begin{matrix} & & 2 & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \end{matrix}$
 $\begin{matrix} & & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \end{matrix}$
 $\begin{matrix} & & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 & 2 \end{matrix}$

**6.6. Conjectured tableaux rank function of content (6, 4).**

$$G_{6,4}^q = 2q^4Q_{10} + 2q^3Q_{9,1} + 2q^2Q_{8,2} + 2qQ_{7,3} + Q_{6,4}$$

 $\begin{matrix} & & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$
 $\begin{matrix} & & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 2^* \end{matrix}$
 $\begin{matrix} & & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \end{matrix}$
 $\begin{matrix} & & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \end{matrix}$
 $\begin{matrix} & & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 & 2 \end{matrix}$

**6.7. Conjectured tableaux rank function of content (7, 2, 1).**

$$G_{7,2,1}^q = 4q^4Q_{10} + (4q^3 + 2q^2)Q_{9,1} + (4q^2 + 2q)Q_{8,2} + 2qQ_{7,3} + Q_{7,2,1}$$

 $\begin{matrix} & & 3 \\ & & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$
 $\begin{matrix} & & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3^* \end{matrix}$
 $\begin{matrix} & & 2 & 2 & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$
 $\begin{matrix} & & 2 & 3^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* \end{matrix}$
 $\begin{matrix} & & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 \end{matrix}$
 $\begin{matrix} & & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2^* & 2 & 2 \end{matrix}$

### 6.8. Conjectured tableaux rank function of content (7, 3).

$$G_{7,3}^q = 2q^3Q_{10} + 2q^2Q_{9,1} + 2qQ_{8,2} + Q_{7,3}$$

	2	2	2	
1	1	1	1	1
	2	2		
1	1	1	1	1
	2			*
1	1	1	1	1
	2			2
1	1	1	1	1
	2		*	2
1	1	1	1	1

### 6.9. Conjectured tableaux rank function of content $(8, 2)$ .

$$G_{8,2}^q = 2q^2Q_{10} + 2qQ_{9,1} + Q_{8,2}$$

### 6.10. Conjectured tableaux rank function of content $(9, 1)$ .

$$G_{9,1}^q = 2qQ_{10} + Q_{9,1}$$