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Symmetries of the k-bounded partition lattice

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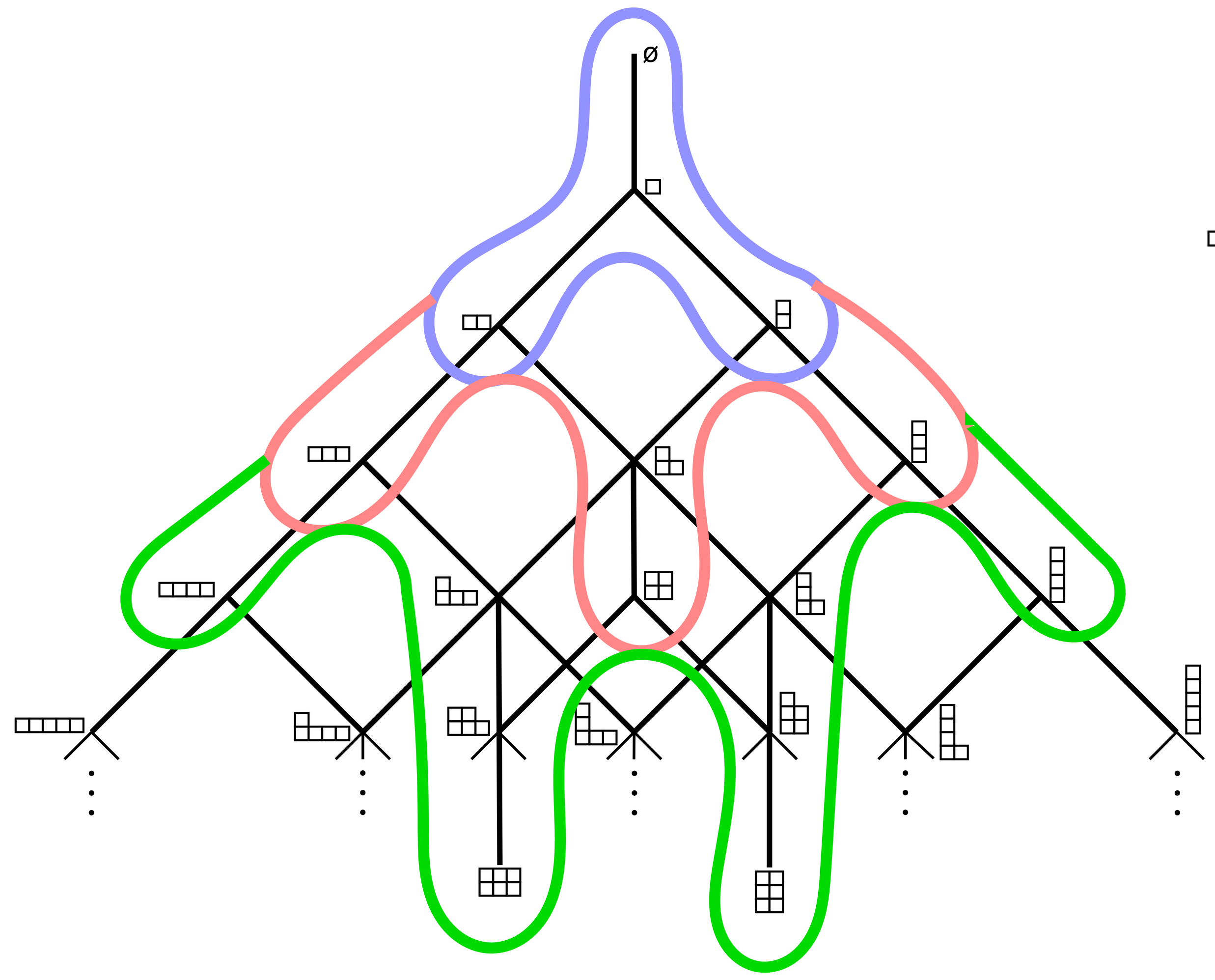
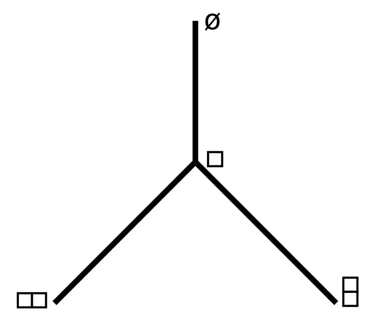
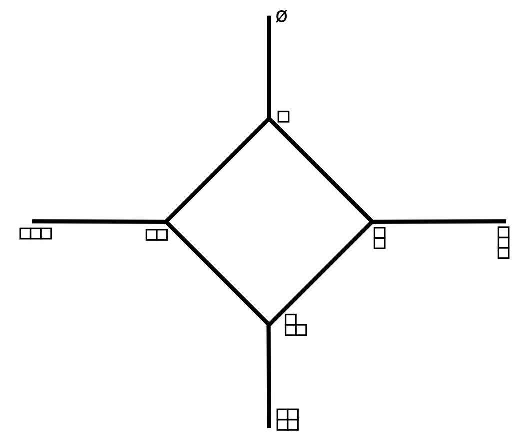
Suter's cyclic symmetry in Young's lattice

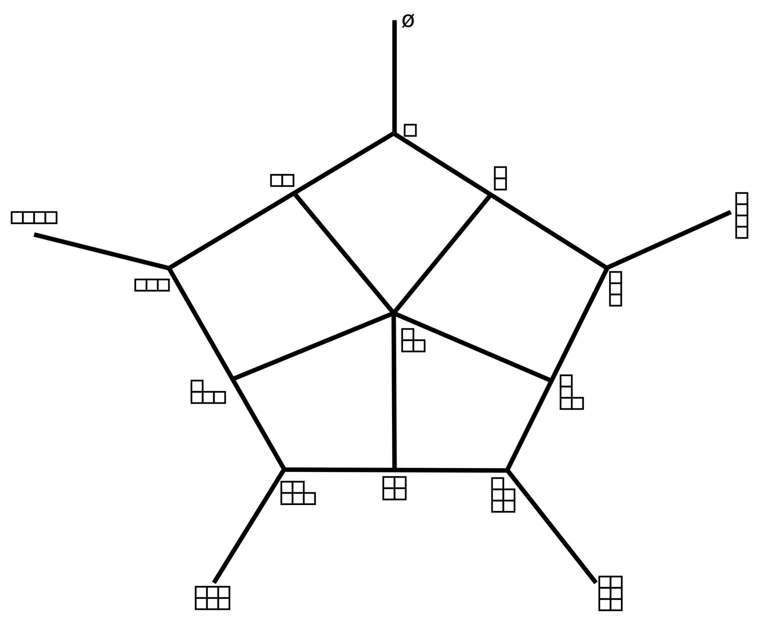




$k = 2$



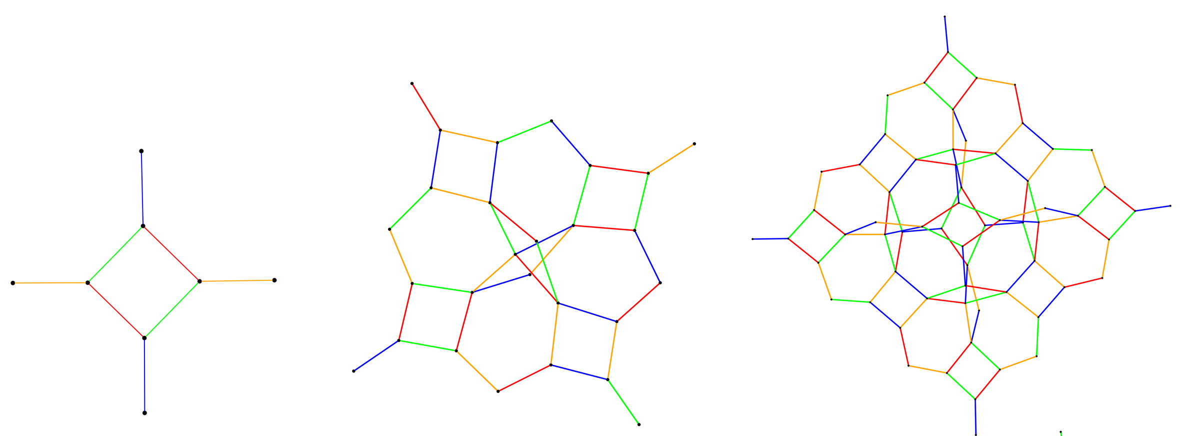
$k = 3$



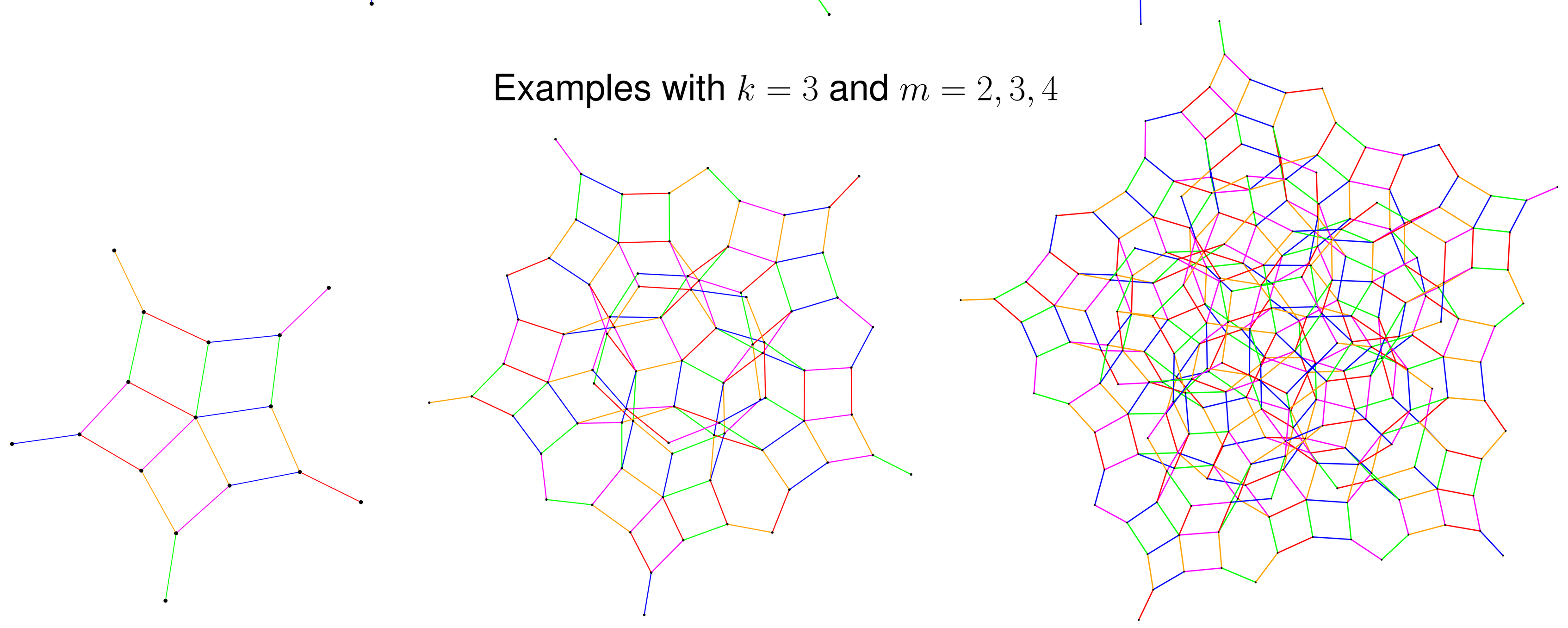
$k = 4$

Proposition 1. *(Suter, 2002) The partitions which are contained in a rectangle whose hook is k forms a finite sub-poset of Young's lattice and this sub-poset has $k+1$-cyclic symmetry.*

Examples of generalized Suter symmetry

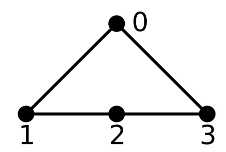


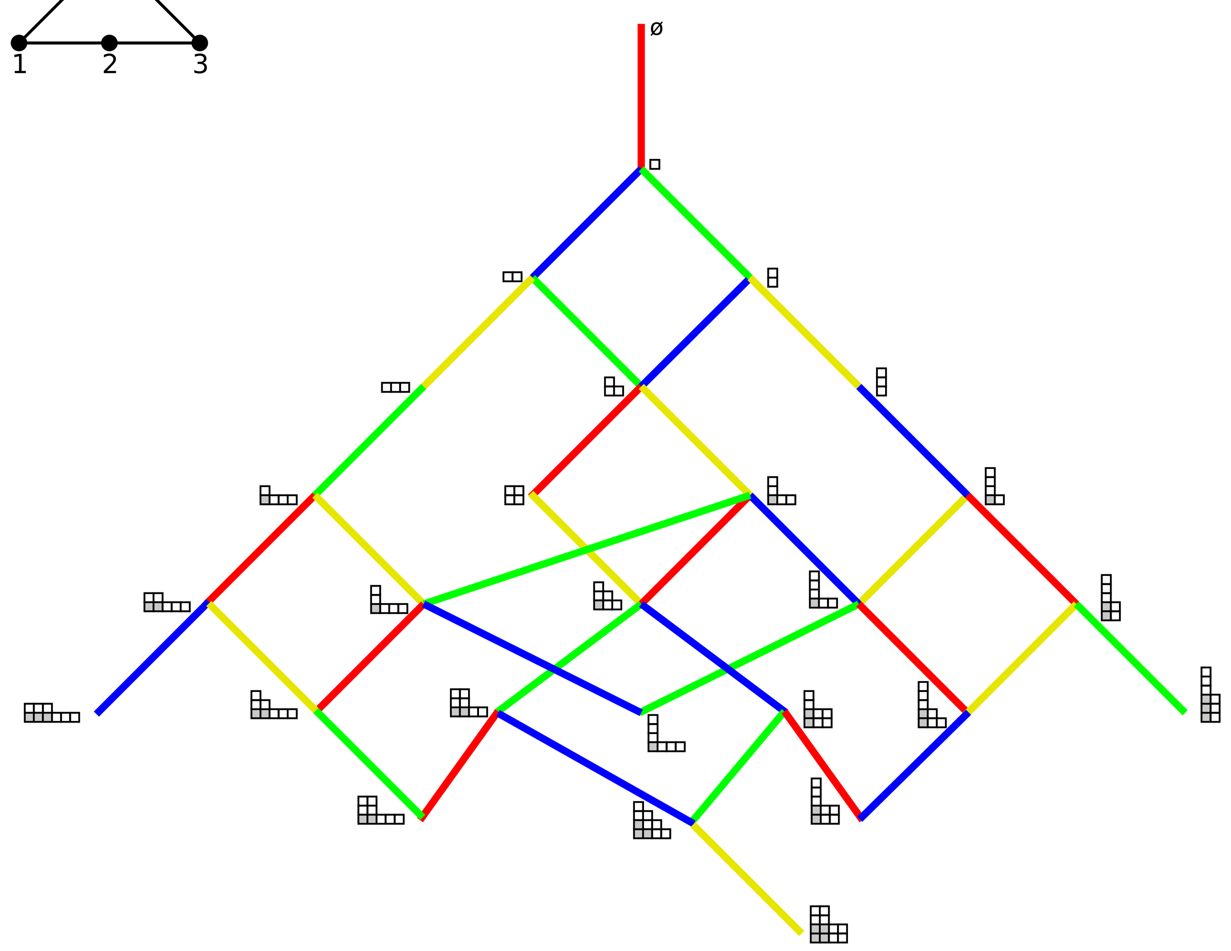
Examples with $k = 3$ and $m = 2, 3, 4$

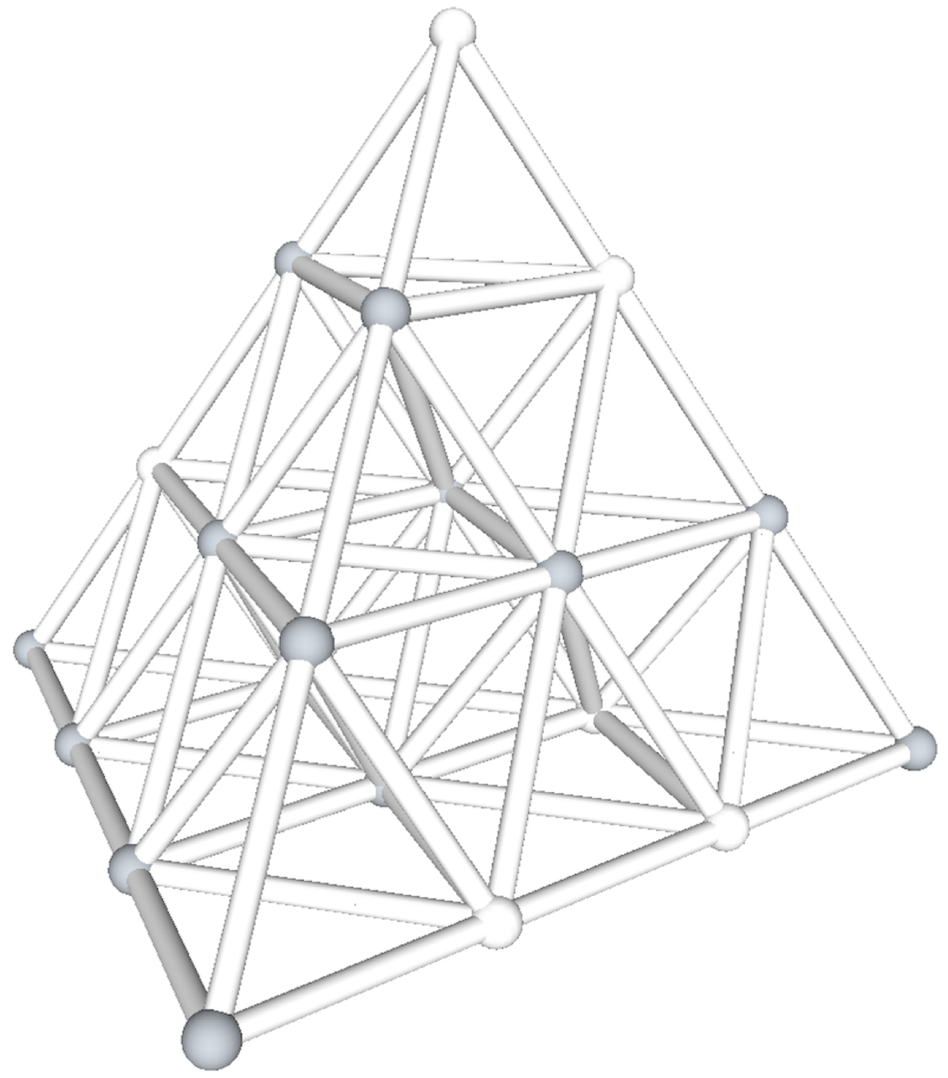


Examples with $k = 4$ and $m = 2, 3, 4$

Weak order on $k+1$-cores and alcoves in the fundamental chamber of $\tilde{A}_k$







Theorem 2. *The alcoves in an m-dialation of the fundamental alcove form a poset isomorphic to the poset of $k+1$-cores contained in a concatenation of $m-1$ different rectangles with a k-hook.*

Here we have shown the $m = 3$ dialation of the $\tilde{A}_3$ fundamental chamber. Suter's symmetry is an $m = 2$ dialation. The cyclic symmetry that we observe on this poset is inherited from the cyclic symmetry of the fundamental alcove and the affine Dynkin diagram of type A.

More precise geometric picture, Connection to k-Schur functions and cyclic seiving

Let $\alpha_i = (\underbrace{0,\dots,0}_{i-1}, 1, -1, \underbrace{0,\dots,0}_{k-i-1})$ for $1 \leq i \leq k$ be the simple roots for the finite root system of type A. Also set $\phi = \alpha_1 + \alpha_2 + \cdots + \alpha_k$ and define $H_{\alpha,r}$ be the set of vectors such that $\langle \alpha, v \rangle = r$ (these are the hyperplanes perpendicular to α at a distance of r from the origin). The m-dilation of the fundamental alcove is the area bounded by the planes $H_{\alpha_i,0}$ and $H_{\phi,m}$.

Proposition 3. *The $k+1$-cores indexed by a concatenation of $m-1$ maximal rectangles are in bijection with the alcoves that share a face with $H_{\phi,m}$ and a vertex with $H_{\phi,m-1}$ and are a translation of the fundamental alcove.*

There are close connections between this picture and k-Schur functions of Lapointe-Lascoux-Morse where elements indexed by maximal rectangles are known to be special elements of the ring.

Nathan Williams has identified an action on words of length k in the alphabet $\{0, 1, 2, \dots, m-1\}$ and a relation that defined a poset with a cyclic sieving phenomenon. Along with Hugh Thomas, they have shown that there is a bijection between the poset of words and the m dilation of the fundamental alcove. In summary, we know that following sets are in bijection:

$k+1$ cores contained in a concatenation of $m-1$ maximal rectangles
k bounded partitions also contained in a concatenation of $m-1$ maximal rectangles
alcoves in the m dilation of the fundamental alcove
words of length k in the alphabet $\{0, 1, 2, \dots, m-1\}$
words of length $k+1$ in the alphabet $\{0, 1, 2, \dots, m-1\}$ that sum to 0 $(mod\ m)$