TALK in MICHIGAN

$\mathbb{Q}\langle H_1, H_2, \ldots \rangle = NSYM$ ⇣ HOPF DUALITY (ASK CARITO)

$\downarrow$

$\mathbb{Q}[h_1, h_2, \ldots] \simeq SYM \hookrightarrow QSYM$

H_λ

$\langle h_\lambda, m_\mu \rangle = \delta_{\lambda\mu}$

$\langle S_\lambda, S_\mu \rangle$

"$s_\lambda = \sum x^T$"

$m_\lambda = \sum$ ORBIT OF $\{x^\lambda\}$

$= \sum_{\alpha \sim \lambda} M_\alpha \longmapsto \sum_{\alpha \sim \lambda} M_\alpha$

$M_\alpha = \sum$ ORBIT OF x^α

	H_i	NSYM	α: COMPOSITIONS
$\downarrow$	$\downarrow$	$\downarrow \chi$	$\downarrow$
S_λ	h_i	SYM	λ: PARTITIONS

$\chi(s_\alpha) = s_{\mathrm{sort}(\alpha)}$ — DUAL TO QUASI-SCHUR (M-L-vW-HAGLUND)

$\chi(Ш_\alpha) = \begin{cases} s_\lambda & \text{if } \alpha = \lambda \\ 0 & \text{otherwise} \end{cases}$ — SHIN BASIS (Z & STUDENTS)

$\chi(I_\alpha) = s_\alpha = \det(h_{\alpha_i + j - i})$ — IMMACULATE BASIS (BBSSZ)

$= \begin{cases} \pm S_\lambda & \text{JT-SORT}(\alpha) = \lambda \\ 0 & \text{otherwise} \end{cases}$

How to define I_α $\qquad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_\ell)$

$$I_\alpha = \text{"Det"}\left(H_{\alpha_i + j - i}\right) = \sum_{\sigma \in S_\ell} H_{\alpha + \sigma - Id}$$

LOOK AT $I_{12} = \begin{Vmatrix} H_1 & H_2 \\ H_1 & H_2 \end{Vmatrix}$

- WHY IS THIS A BASIS?

$I_\alpha = H_\alpha +$ HIGHER TERM (LEX)

- WHY "IMMACULATE"?

(in SYM) $B_m(s_\lambda) = s_{(m,\lambda)} \Rightarrow \boxed{s_\lambda = B_{\lambda_\ell} \cdots B_{\lambda_2} B_{\lambda_1}(1)}$

$$B_m = \sum_{i \geq 0} (-1)^i h_{m+i} e_i^\perp$$

~~$\langle f, e_i g \rangle = \langle e_i f, g \rangle$~~

$\langle e_i^\perp f, g \rangle = \langle f, e_i g \rangle$

$e_i = m_{\underbrace{11\cdots1}_{i}}$

in (NSYM — QSYM)

$E_i = M_{\underbrace{11\cdots1}_{i}}$ $\qquad \langle H_\alpha, M_\beta \rangle = \delta_{\alpha,\beta}$

$\langle E_i^\perp \psi, \phi \rangle = \langle \psi, E_i \phi \rangle$

$$\mathbb{B}_m = \sum_{i \geq 0} (-1)^i H_{m+i} \ldots$$

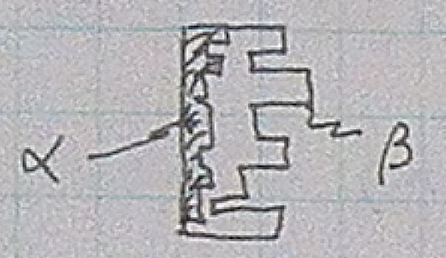
THM $\quad \mathbb{B}_m(I_\alpha) = I_{(m,\alpha)} \Rightarrow \mathbb{B}_{\alpha_\ell} \cdots \mathbb{B}_{\alpha_1}(1) = I_\alpha$

OTHER NICE PROPERTIES ?

- (RIGHT PIERI RULE)

<u>THM</u> $$I_\alpha H_k = \sum_{\beta} I_\beta$$

- $|\beta| - |\alpha| = k$
- $\ell(\beta) \leq \ell(\alpha)+1$
- $\alpha_i \leq \beta_i$



- <u>IMM - TABLEAUX</u>

<u>THM</u> $$H_\beta = \sum_{\gamma} K_{\alpha,\beta} I_\alpha$$

$$K_{\alpha,\beta} = \{ T : \beta \longrightarrow \mathbb{N} \mid \}$$

- "Filling of β"
- WEAKLY INCREASING IN ROW
- <u>FIRST</u> COLUMN STRICTLY INC.
- CONTENT α

$$\left[I^*_\alpha = \sum K_{\alpha\beta} M_\beta \right]$$

P-PARTITION

- L-R RULE

<u>THM</u> $$I_\alpha I_\lambda = \sum c^\beta_{\alpha,\lambda} I_\beta$$

$$C^\beta_{\alpha\lambda} = |\{ T : \beta/\alpha \longrightarrow \mathbb{N} \}|$$

- SKEW-IMMACULATE TABLEAUX
- CONTENT λ
- YAMANOUCHI

~~GEOMETRIC (POLYTOPE) INTERP~~

- <u>COR</u> $$\boxed{C^\beta_{\alpha,\lambda} = C^{\beta+\nu}_{\alpha+\nu,\lambda}} \quad \forall\, \ell(\nu) \leq \ell(\alpha)$$

SPECIAL HERE !

- GEOMETRIC (POLYTOPE) INTERPRETATION :

<u>COR</u> VARIABLES $\{ x_{ij} \}_{0 \leq i \leq j \leq \underbrace{\max\{\ell(\beta), \ell(\alpha), \ell(\lambda)\}}_{m}} \subseteq \mathbb{R}^{\binom{m+1}{2}}$

HYPER PLANE & HALF SPACES

(0) $x_{00} = 0$

(1) $x_{0j} - x_{0\,j-1} = \alpha_j$

(2) $x_{jj} - x_{j-1,j-1} = \beta_j$

(3) $x_{im} - x_{i-1,m} = \lambda_i$

} ~~HYPER~~ HYPERPLANE

(4) $x_{ij} - x_{i\,j-1} \geq x_{i-1\,j} - x_{i-1\,j-1}$

(5) $x_{ij} - x_{i-1\,j} \geq x_{i+1\,j+1} - x_{i\,j+1}$

(6) $N(x_{i-1,j} - x_{i-1\,j-1}) \geq x_{i\,j+1} - x_{ij}$

} HALF-SPACE

$$\boxed{\#\begin{pmatrix} \text{INTEGER COORDINATE VECTOR} \\ \text{INSIDE REGION (POLYTOPE)} \\ \text{ABOVE} \end{pmatrix} = C^\beta_{\alpha\lambda}}$$

WHAT NEXT ?

- PEAK ALGEBRA
- HALL-LITTLEWOOD
- MACDONALD.