- For the functions $\mathbb{H}_\alpha^{qt}$ we know that

$\mathbb{H}_\alpha^{q1}$ is a product, can $\mathbb{H}_\alpha^{qt}$ be seen as a composition of creation operators and in particular is $\mathbb{H}_\alpha^{qt}$ equal to a composition of operators

$$\widetilde{V}_r^{\,t}$$

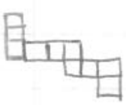
where V_r is an operator which does not involve a t?

Note: $\widetilde{V}_r^{\,t}\big|_{t=1}$ will be multiplication by $V_r(1)$

Conjecture above $\Longrightarrow \mathbb{H}_\alpha^{q1} = V_{r_1}(1)\, V_{r_2}(1) \cdots V_{r_k}(1)$

$$\mathbb{H}_\alpha^{q1} = \mathbb{H}_{(\alpha_1)}^{q(\sum_{i\geq 1}\alpha_i)}\, \mathbb{H}_{(\alpha_2)}^{q(\sum_{i\geq 2}\alpha_i)} \cdots \mathbb{H}_{(\alpha_k)}^{q(0)}$$

where $\mathbb{H}_{(m)}^{q(i)} = \sum_{\beta \models m} q^{(\ell(\beta)-1)i + n(\bar{\beta})}\, s_\beta$

(Nantel:  $V_r(f) = \mathbb{H}_r^{q(d)} f$, $\deg f = d$ $\quad V_r = \oplus V_r^{(i)}$)

$$V_r^{(i)}(1) = \mathbb{H}_r^{q(i)}$$

- Find an $H_n(0)$-module model for the γ-ribbon functions.

- The γ-ribbon functions defined as

$$s_\alpha^{(\gamma)} = \sum_{\substack{\beta \geq \alpha \\ D(\alpha)/D(\beta) \subseteq D(\gamma)}} t^{c(\alpha,\beta^c)}\, s_\beta$$

What is $s_\alpha^{(r)} \cdot s_\beta^{(r)} = ?$

What is $\Delta(s_\alpha^{(r)}) = ?$

According to analogous theory in commutative case

$$s_\alpha^{(r)} \cdot s_\beta^{(r)} \Big|_{t=1} \text{ should have a nice formula}$$

$\Delta(s_\alpha^{(r)})$ should be nice

and there should be a product $*$ which is not normal product & involves a parameter t.
such that $*|_{t=1}$ = usual product. → if use $\cdot$ the product goes out (it couldn't be written in r-ribbon)

$$s_\alpha^{(r)} * s_\beta^{(r)} = \text{some "nice" formula}$$

conjecture: $H_\alpha^t * H_\beta^t = H_{\alpha\cdot\beta}^t + t^{\alpha} H_{\alpha|\beta}^t$

<u>François</u>

$H_n(0)$-module for NC HL functions

$$H_\alpha(A;t) = \sum_{\beta \geq \alpha} t^{c(\alpha,\beta^c)} R_\beta(A)$$

$$\widehat{H}_\alpha(A;t) = t^{n(\alpha)} \sum_{\beta \geq \alpha} t^{-c(\alpha,\beta^c)} R_\beta(A)$$

$$\boxed{\widetilde{H}_\alpha(A;t) = \sum_{\beta \geq \alpha} t^{c(\alpha,\beta)} R_\beta(A)}$$

<u>Question</u>: Find an $H_n(0)$-module V_α such that

$\widetilde{H}_\alpha(A;t)$ is the <u>graded</u> characteristic

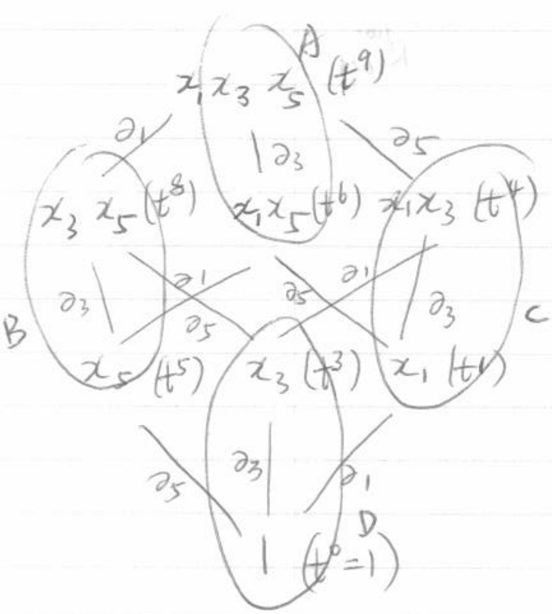
$V_\alpha \subset \mathbb{C}[x_1, \dots, x_{|\alpha|}]$

<u>Answer</u>: $\alpha \longrightarrow Des(\alpha)$

$$X_{Des(\alpha)} = x_{\alpha_1} x_{\alpha_1+\alpha_2} \cdots x_{\alpha_1+\cdots+\alpha_{\ell(\alpha)-1}}$$

$\alpha = (1221) \quad Des(\alpha) = (135)$

V_{1221}



$deg(x_i) = i$

R_{1221}

$R_{321} \quad R_{141} \quad R_{123}$

$R_{51} \quad R_{33} \quad R_{15}$

R_6

Action of $H_n(0)$ on $\mathbb{C}[x_1, \dots, x_{|\alpha|}]$

$$T_i X_S^\delta = \begin{cases} -X_S^\delta & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases}$$

$$\widetilde{H}_\alpha(A;t) = \sum_{\alpha \le \beta \le \gamma} t^{c(\alpha,\beta)} R_\beta^{(\gamma)}(A; 1/t) = \sum_{\alpha \le \beta \le \gamma} t^{n(\beta)} R_\beta^{(\gamma)}(A; 1/t)$$

Mike vs François

$\boxed{S = R}$

$$R_\alpha^{(\gamma)}(A;t) = \sum_{\alpha \le \beta \le \gamma} (-1)^{\ell(\alpha)-\ell(\beta)} t^{c(\alpha,\beta^c)} H_\beta(A;t)$$

$$\widetilde{H}_{1221}(A;t) = t^6 \overset{A}{R_{1221}^{(33)}(X; 1/t)} + t^5 \overset{B}{R_{321}^{(33)}(X; 1/t)} + t^1 \overset{C}{R_{123}^{(33)}(X; 1/t)}$$
$$+ t^0 \overset{D}{R_{33}^{(33)}(X; 1/t)}$$

$$\widetilde{R}_{\alpha}^{(\gamma)}(A;t) = t^{n(\gamma)} R_{\alpha}^{(\gamma)}(A;1/t)$$

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Remark 1 :

Nantel :

$$\begin{tikzcd}[column sep=small]
 & x_1\underline{x_3}x_7\underline{x_8}x_9 \arrow[dl, dash] \arrow[dr, dash] & \\
x_1x_7\underline{x_8}x_9 \arrow[dr, dash] & & x_1\underline{x_3}x_7x_9 \arrow[dl, dash] \\
 & x_1x_7x_9 &
\end{tikzcd}$$

$$\begin{tikzcd}[column sep=small]
 & \widetilde{R}^{(352)}_{(124111)} & \\
\widetilde{R}^{(82)}_{(16111)} & & R^{(37)}_{(12421)} \\
 & R^{(10)}_{(1621)} &
\end{tikzcd}$$

Nov. 23, 2007 Fields 3:30pm

Mike

$$\Delta(\mathcal{S}_\alpha) = \sum_{S \subseteq [n]} \mathcal{S}_{\alpha(w_S)} \otimes \mathcal{S}_{\alpha(w_{S^c})}$$, where w is a permutation with $D(w) = D(\alpha)$ and $w_S = w_{s_1} \cdots w_{s_i}$ for $s_i \in S$ and $\alpha(u)$ = the composition with descent set same as descent set of u.

$\underline{7}3\underline{4}5\underline{6}2\underline{1}$ $741/3562$

$\otimes$

$$\Delta(\mathcal{S}_{(22)}) = \mathcal{S}_\phi \otimes \mathcal{S} + \mathcal{S}_\square \otimes (2\mathcal{S} + 2\mathcal{S})$$

$$+ \mathcal{S} \otimes 4\mathcal{S}$$

$$+ \mathcal{S} \otimes 2\mathcal{S}$$

$$+ (2\mathcal{S} + 2\mathcal{S}) \otimes \mathcal{S}_\square$$

$$+ \mathcal{S} \otimes \mathcal{S}_\phi$$

3 4
1 2

$w = 3412$

ring of NSym $\Delta(\mathcal{S}_\alpha)\big|_{\mathcal{S}_\beta \otimes \mathcal{S}_\gamma} = F_\beta F_\gamma \big|_{F_\alpha}$ ring of QSym

$$F_\square \cdot F = F + F + F + F$$

1 423 1423 4123 4213 4231

$$\Delta(\mathcal{S}_\alpha) = \sum_{k=0}^{n} \mathcal{S}_{\alpha(w|_{1\cdots k})} \otimes \mathcal{S}_{\alpha(w|_{k+1,\cdots,n})}$$

$$\Delta(s_{\ydiagram{2,1+2}}) = s_\phi \otimes s_{\ydiagram{2,1+2}} + s_{\square} \otimes s_{\ydiagram{2,1+1}} + s_{\square} \otimes s_{\ydiagram{3}}$$

$$+ s_{\square} \otimes s_{\ydiagram{1,2}} + \cdots$$

$$F_{\square} \cdot F_{\ydiagram{2,1+1}} = F_{\ydiagram{3,2+1}} + F_{\ydiagram{1,2,1+1}} + F_{\ydiagram{2,1+2}} + F_{\ydiagram{2,1+1,1+1}}$$

$1 \quad 342 \qquad 1342 \qquad 3142 \qquad 3412 \qquad 3421$

$$\begin{aligned}\Delta(s_{\ydiagram{2,1+2}}) = {} & s_\phi \otimes s_{\ydiagram{2,1+2}} \\ & + s_{\square} \otimes \left(s_{\ydiagram{2,1+1}} + s_{\ydiagram{3}} + s_{\ydiagram{1,2}}\right) \\ & + s_{\ydiagram{2}} \otimes \left(2 s_{\ydiagram{2}} + s_{\ydiagram{1,1}}\right) \\ & + s_{\ydiagram{1,1}} \otimes \left(s_{\ydiagram{2}} + s_{\ydiagram{1,1}}\right) \\ & + \left(s_{\ydiagram{2,1+1}} + s_{\ydiagram{3}} + s_{\ydiagram{1,2}}\right) \otimes s_{\square} \\ & + s_{\ydiagram{2,1+2}} \otimes s_\phi\end{aligned}$$

$$\Delta(s_\alpha) = \sum_{w : D(w) = \alpha} \sum_k \chi\left(w|_{1\cdots k} = \alpha(w|_{1\cdots k})^+\right) \chi\left(w|_{k+1\cdots n} = \alpha(w|_{k+1\cdots n})^+\right) \quad s_{\alpha(w|_{1\cdots k})} \otimes s_{\alpha(w|_{k+1\cdots n})}$$

<u>Nantel</u>: $u = 132$, $u = 231$ $\quad \ydiagram{2,1+1} \quad \alpha(u)^+ = 231$

$$s^{(13)}_{(13)} = s_{(13)} + t\, s_{(4)}$$

$$s^{(13)}_{(121)} = s_{(121)} + t\, s_{(31)}$$

$$\otimes \quad s^{(13)}_{(112)} = s_{(112)} + \cancel{t}\, s_{(22)}$$

$$s^{(13)}_{(1111)} = s_{(1111)} + t\, s_{(211)}$$

<u>François</u> $\Delta\left(s^{(13)}_{(112)}\right) = s \otimes \left(s_{\boxplus} + t\, s_{\boxplus}\right) + s_{\square} \otimes \Big(s_{\boxminus} + t\, s_{\boxplus}$

$$+ (t+1)\, s_{\boxplus} + t\, s_{\square\square\square}\Big)$$

$$+\, s^{(2)}_{\square\square} \otimes \left((t+1)\, s_{\boxminus} + 2t\, s_{\square\square}\right) + \cdots$$

$112 \leftrightarrow \{1, 2\}$

<u>Nantel</u>:

$$NSym \hookrightarrow \bigoplus \mathbb{Z} S_n \circlearrowleft^{*} \qquad R_\alpha \longmapsto \sum_{D(w)=\alpha} w \qquad w^*$$

$$\downarrow \qquad\qquad\qquad\qquad\qquad \downarrow$$

$$QSym \qquad\qquad \Delta w = \sum_k w|_{1\cdots k} \otimes w|_{k+1\cdots n}$$

<u>François</u> $$R_\alpha^{(\gamma)} = \sum_{\substack{\beta \geq \alpha \\ D(\alpha)/D(\beta) \subseteq D(\gamma)}} t^{c(\alpha, \beta^c)} R_\beta$$

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$$s^{(\gamma)}_{\sigma} = \sum_{\substack{w \,:\, D(w) \subseteq D(\sigma) \\ D(w) \setminus D(\sigma) \subseteq D(\gamma)}} t^{c(w,\sigma^{-1}w_0)^{?}}\, w \qquad \text{conjugation by } w_0$$

$$\Delta\left(s^{(13)}_{(112)}\right) = s \otimes s^{(13)}_{(112)} + s_1 \otimes \left(s^{(12)}_{(111)} + (t+1)\, s^{(3)}_{(12)} + s^{(3)}_{(3)}\right)$$
$$+ \cdots$$