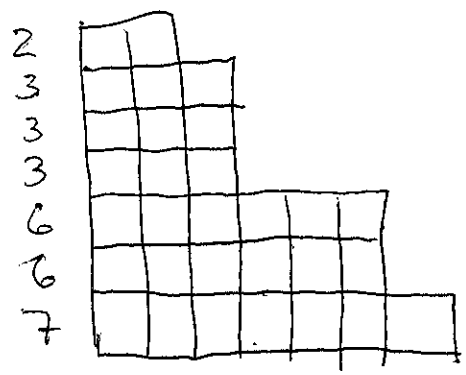
One property that I would like to highlight is a rule for taking the product of h_r and s_λ. It is called the "Pieri-rule"

To describe this rule properly we want to picture a partition as a set of boxes stacked in rows. The rows will all be left justified and there are λ_i boxes in each row.

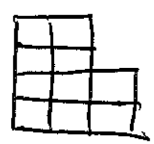
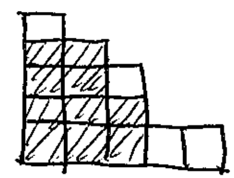
Example $\lambda = (7,6,6,3,3,3,2)$



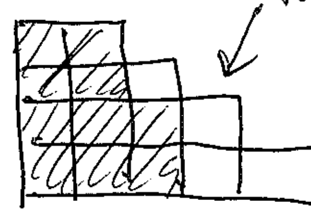
Young diagram OR Ferrer's diagram for the partition λ.

We say that a partition μ differs from λ by a horizontal strip if the diagram for μ contains the diagram for λ and if all the extra cells in μ that are not in λ are in different columns

Examples $\lambda = (3,3,2,2)$ $\mu = (5,3,3,2,1)$ is a horizonta strip

λ  μ 

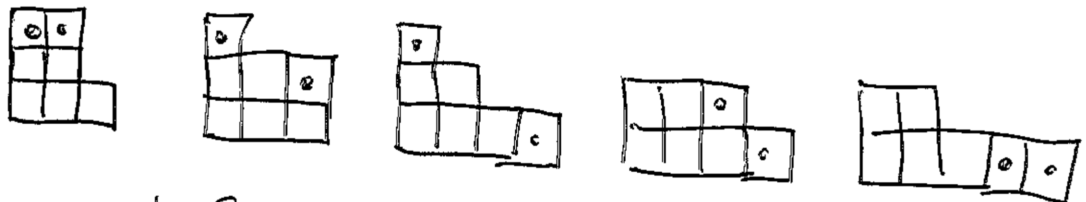
$\lambda = (3,3,2,2)$ $\mu = (5,4,3,2)$ is not a horizontal strip



two cells in same column.

The Pieri rule says that h_r times s_λ is the sum over all Schur functions indexed by partitions μ that differ from λ by a horizontal strip of size r (all with coefficient equal to 1).

Example:



$$h_2 s_{32} = s_{322} + s_{331} + s_{421} + s_{43} + s_{52}.$$

Note this would be hard to do just using the definition

$$h_2 \cdot s_{32} = h_2(h_{32} - h_{41}) = h_{322} - h_{421} = ?$$

This formula is not that easy to prove either...
I probably won't do it in a video.