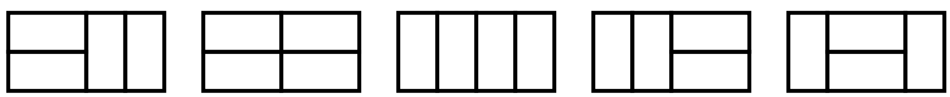
DISCUSSION FOR NINTH TUTORIAL

DATE: FRIDAY MAR 4 (LCT01), MONDAY MAR 7 (LBT01), FRIDAY MAR 11 (LCT02), MONDAY MAR 14 (LBT02 & LBT03)

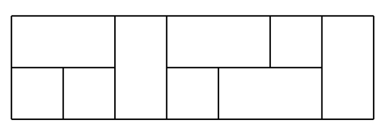
The following is partially taken from Konhauser, Velleman, and Wagon, *Which Way Did the Bicycle Go? . . . and Other Intriguing Mathematical Mysteries.*

Problem: Consider the number of ways tilings of a $2 \times n$ rectangle using only 1×2 shaped tiles placed horizontally or vertically as in the following example when $n = 4$.



Compute the number of tilings by example for $n = 1, 2, 3, 5, 6$ and explain why the number of tilings is related to a sequence you have seen before and why they satisfy the same recursive formula.

Next consider the diagram below with a tiling of a 2×7 rectangle with 1×1 and 1×2 tiles (singletons and dominoes; dominoes may be placed horizontally or vertically). How many such tilings of a 2×7 grid are there? More generally, how many tilings of a $2 \times n$ grid are there?



Let a_n be the number of tilings of a $2 \times n$ grid using 1×1 and 1×2 tiles so that the right most column is occupied by singletons, let b_n be the number of tilings with one singleton and one doubleton in the right most column, and let c_n be the number of other tilings (two horizontal doubletons or one vertical doubleton). The problem asks for $a_7 + b_7 + c_7$. When one appends another column to the right side, forming a $2 \times (n + 1)$ grid, either one can add 2 singletons or a vertical doubleton, or one can change any sigletons in the nth column to doubletons. You should explain clearly how this yields three recurrence relations:

$$a_{n+1} = a_n + b_n + c_n$$
$$b_{n+1} = 2a_n + b_n$$
$$c_{n+1} = 2a_n + b_n + c_n$$

Use these recurrence relations to obtain a general method for computing a_n, b_n and c_n. Use this formula to compute $a_7 + b_7 + c_7$. An ideal solution should explain how one might compute $a_{50} + b_{50} + c_{50}$ (a 25 digit number).