

## HOMEWORK ASSIGNMENT NO. 5

DATE GIVEN: FEBRUARY 27, 2012 DUE: MARCH 19, 2011

- (1) Let  $n$  be an integer. Justify the following statements.
  - (a) The last digit of  $n$  is even if and only if  $n$  is divisible by 2.
  - (b) The last two digits of  $n$  are divisible by 4 if and only if  $n$  is divisible by 4.
  - (c) The last three digits of  $n$  are divisible by 8 if and only if  $n$  is divisible by 8.
  - (d) The last  $k$  digits of  $n$  are divisible by  $2^k$  if and only if  $n$  is divisible by  $2^k$ .

Recall that we call a function  $f : A \rightarrow B$  'injective' or '1-1' if for all  $x, y \in A$ , if  $f(x) = f(y)$ , then  $x = y$ . And we call a function 'surjective' or 'onto' if for every  $y \in B$ , there is an  $x \in A$  such that  $f(x) = y$ .

- (2) Consider the function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  where  $f(x) = x^3 - 4x$ 
  - (a) Is  $f$  injective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be injective.
  - (b) Is  $f$  surjective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be surjective.
- (3) Consider the function  $g : \mathbb{R} \rightarrow \mathbb{R}$  where  $g(x) = x^3 - 1$ .
  - (a) Is  $g$  injective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be injective.
  - (b) Is  $g$  surjective? Why or why not? If it is demonstrate or explain why. If not, give an example of where it fails to be surjective.
- (4) Let  $W$  = the set of words in the letters  $a$  and  $b$  (the empty word counts as a word).

Let  $f_1 : W \rightarrow W$  be defined so that for a word  $w \in W$ ,  $f_1(w)$  be the word found by inserting the letter  $a$  in the middle of  $w$  word  $w$  if  $w$  has even length and putting an  $a$  at the beginning of the word if it has odd length. For instance  $f_1(abbbb) = abbabb$  and  $f_1(baaab) = abaaab$ . If  $w$  is the empty word, then  $f_1(w) = a$ .

  - (a) Is this function onto? Why or why not? Explain completely.
  - (b) Is this function 1-1? Why or why not? Explain completely.