Homework Assignment no. 3

Date: from Tutorial on Wedensday October 18,2017; Due Wednesday, November 8, 2017

Your assignment should include complete sentences and explanations and not just a few equations, tables or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

Note that if a is an integer, then a divides b if $a \neq 0$ and there is an integer k such that $ak = b$. An integer a is a perfect square if there is an integer k such that $a = k^2$.

(1) The statements below are all true, provide a proof of why they are true.
   (a) For integers a and b, $a - b$ is odd if and only if $a + b$ is odd.
   (b) If y is an integer, then $y^3/3 + y^2 - 10y/3 + 2$ is an integer.
   (c) If x and y are positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.
   (d) The product of a rational number and an irrational number is an irrational number.
   (e) The sum of a rational number and an irrational number is an irrational number.

(2) The following statements are either true or false. For each statement that is true give an explanation why. For each statement that is false give a counterexample.
   (a) For integers $m > 0$ and $a, b, c, d \in \mathbb{Z}$, if m divides $a - b$ and m divides $c - d$, then m divides $ac - bd$.
   (b) For $m > 0$ and $a, b \in \mathbb{Z}$, if m divides ab, and m does not divide a, then m divides b.
   (c) For positive real numbers x, y, $\sqrt{xy} \leq \frac{x+y}{2}$
   (d) For every positive integer n, $n^3 - 10n$ is divisible by 3.
   (e) There are rationals r and s such that $\sqrt{3} = r + s\sqrt{2}$.
   (f) The number $\sqrt{3}$ is irrational.
   (g) For every $n > 0$, $n^2 - n + 11$ is prime.

(3) A bank was robbed and Inspector Craig and Sergeant McPherson were on the case trying to establish the guilt or innocence of three suspects Alice, Bob and Carol. The nefarious characters are the only people who could be involved in these bank robberies and at least one of them is guilty. In each case the Inspector and Sergeant establish certain facts.

   Write an argument in words to establish the guilt or innocence of Alice, Bob and Carol. Note that the clues provided may not be sufficient to determine the guilt and innocence of all of the suspects, but should be sufficient to establish the guilt of at least one person.

   Say that we establish that:
   (i) If Alice was guilty, then she had an accomplice.
   (ii) If Bob is innocent, then so is Carol.
   (iii) If exactly two are guilty then Alice is one of them.
   (iv) If Carol is innocent, then so is Bob.

   One way to organize your information is to translate each of the clues to a truth valued sentence *if ... then* and the propositions: A representing the statement "Alice is guilty," B representing the statement "Bob is guilty" and C representing "Carol is guilty." Create a truth table establishing the truth values of the clues in terms of the truth values of A, B and C.