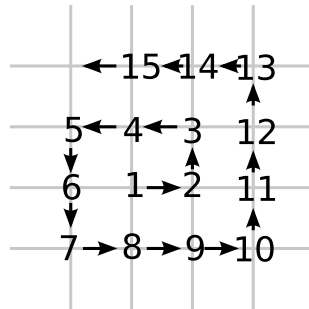


## HOMEWORK #5

DATE: JANUARY 9, 2018 : DUE: JANUARY 31, 2018

The following problem is (roughly) from Thinking Mathematically Second Edition p. 125 (Number Spirals).

Say that you want to number the vertices of a grid using a spiral pattern as you see in the picture below. Say that the number 1 is on the origin  $(0,0)$  then 2 is placed on the vertex at  $(1,0)$ , 3 is placed at  $(1,1)$ , 4 at  $(0,1)$ , 5 at  $(-1,1)$ , 6 at  $(-1,0)$ , and subsequent numbers follow the pattern below.



One question I might ask is, “what are the coordinates of the number 411?” The way you might proceed to find the answer is to draw a grid sufficiently large that it contains enough squares to fill in the numbers 1 through 411 and proceed to draw a big spiral. That is an ok answer for that particular value, but it does nothing to help me find the coordinates of 17,411 or 17,300,411. One way to come up with a method of finding where these numbers appear is to consider some special cases:

- Where are the square numbers? even numbers? odd numbers? multiples of 3?
- What numbers are on the corners of the form  $(a, a)$ ? of the form  $(a, -a)$ ? (for  $a \in \mathbb{Z}$ )
- What numbers appear on particular rows or columns?

The main question you should try to answer is “given a positive integer  $n$ , what are the coordinates where that integer is placed?” This should be a formula or a procedure for finding those coordinates that should apply to all integers and in particular work on the examples above.