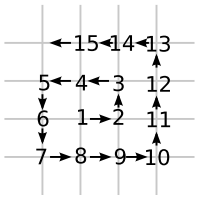
PRACTICE PROBLEMS

Provide a full explanation of the

(1) Give an example of a polynomial with real coefficients of degree 3 with
   (a) zero real roots
   (b) one real root
   (c) two real roots
   (d) three real roots

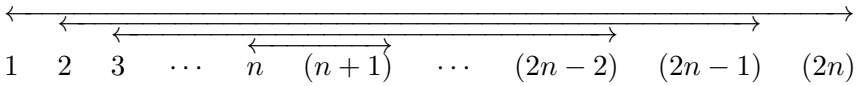
(2) Give an example of a polynomial with real coefficients of degree 4 with
   (a) zero real roots
   (b) one real root
   (c) two real roots
   (d) three real roots
   (e) four real roots

(3) find a polynomial $p(x)$ with real coefficients such that $p(r)$ = the value on the ray traveling west from the 1 in the picture below (e.g. $p(1) = 1, p(2) = 6, p(3) = 19$, etc.):



What are the values of $p(0), p(-1), p(-2)$, etc.? Why?

(4)



Consider the natural numbers from 1 to $2n$. Pair off these numbers as above, 1 and $(2n)$, 2 and $(2n-1)$, 3 and $(2n-2)$, ..., n and $(n+1)$, and evaluate the products of the pairs, $1 \times (2n)$, $2 \times (2n-1)$, $3 \times (2n-2)$, ..., $n \times (n+1)$. Prove that for no value of n are two of these n products equal.