Practice

Use complete sentences and explanations.

Note that if a and b are integers and $a \neq 0$, then a divides b if there is an integer k such that $ak = b$.

Let a, b and c be integers. Each of the following statements is either true or false. For each of them, if the statement is true provide a complete proof (full sentences). If the statement is false, provide a counterexample and try to correct the statement.

(1) If a divides b and b divides c, then a divides c.

(2) If a divides b, then a divides bc.

(3) If a divides b and b divides a, then $a = b$.

(4) If $a \neq 0$, then $a|a$.

(5) If a divides b^2, then a divides b.

(6) If a divides $b + c$ and a divides b, then a divides c.

(7) If a divides bc and a does not divide b, then a divides c.

(8) If a divides b and a divides c, then a divides $b + c$.

(9) If ab divides c, then a divides c.