Show that

$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} .$$

Let n be a positive integer, and $a_i \geq 1$, for $i = 1, 2, \ldots, n$. Show that

$$(1+a_1)(1+a_2)\cdots(1+a_n) \geq \frac{2^n}{n+1}(1+a_1+a_2+\cdots+a_n) .$$

Show that for $n > 0$,

$$2!4!\cdots(2n)! \geq ((n+1)!)^n$$

Show that for $n \geq 1$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1$$

For all x in the interval $0 \leq x \leq \pi$, prove that

$$|sin\ nx| \leq nsin\ x$$

where n is a non-negative integer.

Let $f_0(x) = 1/(1-x)$, and define $f_{n+1}(x) = xf_n'(x)$. Prove that $f_{n+1}(x) > 0$ for $0 < x < 1$.

Show that for $n \geq 1$,

$$1 + 2\cos x + 2\cos 2x + 2\cos 3x + \cdots + 2\cos nx = \sin((2n+1)x/2)/\sin(x/2)$$

Show that

$$(\cos x + \cos 2x + \cdots + \cos nx)\sin\left(\frac{x}{2}\right) = \cos\left(\left(\frac{x}{2}\right)(n+1)\right)\sin\left(\frac{nx}{2}\right)$$