BAD PROOFS BY INDUCTION

Proposition 1. *For all $n \geq 0$,*

$$\frac{d}{dx}(x^n) = 0$$

Proof. Base case: $n = 0$,

$$\frac{d}{dx}(x^0) = \frac{d}{dx}(1) = 0$$

Assume that $\frac{d}{dx}(x^m) = 0$ for all $0 \leq m \leq n$, then

$$\begin{aligned}
\frac{d}{dx}(x^{n+1}) &= \frac{d}{dx}(x^n \cdot x) \\
&= x \cdot \frac{d}{dx}(x^n) + x^n \cdot \frac{d}{dx}(x) \qquad \text{by the product rule} \\
&= x \cdot 0 + x^n \cdot 0 \qquad \text{by the inductive assumption} \\
&= 0
\end{aligned}$$

Hence by the principle of mathematical induction $\frac{d}{dx}(x^n) = 0$ for all $n \geq 0$. □

Proposition 2. *For all $n \geq 2$,*

$$2^n < n!$$

Proof. Base case: $2^1 \leq 2!$.

Assume that $2^n < n!$, for some fixed n where $1 \leq n$. Then, $2 \leq n+1$ and

$$2^{n+1} = 2(2^n) < 2(n!) \leq (n+1)n! = (n+1)! \ .$$

Hence, by the principle of mathematical induction $2^n < n!$ for all $n \geq 2$. □

Note: $2^2 \not< 2!$

Proposition 3. *Let* $a_n = 3a_{n-1} - n$ *and* $a_0 = 1$*, then for* $n \geq 0$*,*

$$a_n = \frac{3^n + 2n + 3}{4}$$

Proof. Base case: $a_0 = 1$ and $\frac{3^0 + 2 \cdot 0 + 3}{4} = \frac{1+3}{4} = 1$ and are equal.

Assume that $a_n = \frac{3^n + 2n + 3}{4}$ for some fixed n, then

$$\begin{aligned}
a_{n+1} &= \frac{3^{n+1} + 2(n+1) + 3}{4} \\
3a_n - (n+1) &= \frac{3 \cdot 3^n + 2n + 5}{4} \\
3\left(\frac{3^n + 2n + 3}{4}\right) - (n+1) &= \frac{3 \cdot 3^n + 2n + 5 + (4n+4) - (4n+4)}{4} \\
\frac{3 \cdot 3^n + 6n + 9}{4} - (n+1) &= \frac{3 \cdot 3^n + 6n + 9 - (4n+4)}{4} \\
\frac{3 \cdot 3^n + 6n + 9}{4} - \frac{4(n+1)}{4} &= \frac{3 \cdot 3^n + 6n + 9}{4} - \frac{4(n+1)}{4}
\end{aligned}$$

Hence, by the principle of mathematical induction $a_n = 3a_{n-1} - n$ for all $n \geq 0$. ☐

Proposition 4. *For all $n \geq 1$,*

$$\underbrace{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots}_{n\text{ terms}}=\frac{3}{2}-\frac{1}{n} \tag{1}$$

Proof. For $n = 1$, we have $\frac{1}{1\cdot 2} = 1/2$ and $\frac{3}{2} - \frac{1}{1} = 1/2$.

Assume that (1) holds for some fixed n, then

$$\begin{aligned}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)n}\right)+\frac{1}{n(n+1)} &= \frac{3}{2}-\frac{1}{n}+\frac{1}{n(n+1)}\\ &= \frac{3}{2}-\frac{1}{n}+\frac{1}{n}-\frac{1}{n+1}\\ &= \frac{3}{2}-\frac{1}{n+1}\end{aligned}$$

□

Note: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} \neq \frac{3}{2} - \frac{1}{2}$

All of the proofs above have some fault in the argument. Some you will have to go through very carefully to see what goes wrong.

Note: some of these proofs were taken from
https://math.stackexchange.com/questions/1196303/fake-induction-proofs

Other proofs are modified versions of what is in the textbook.

For the proof that all horses are the same color see
https://en.wikipedia.org/wiki/All_horses_are_the_same_color