

ASSIGNMENT #3

DATE: OCTOBER 15, 2018 DUE: OCTOBER 28, 2018

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

- (1) The following statements depend on a variable, x , which will specialize to elements from the set of polygons. Let $R(x)$ be the proposition that “ x is a rectangle,” $Rh(x)$ be the statement that “ x is a rhombus” and $S(x)$ is the proposition that “ x is a square.” Moreover, set $EP(x)$ to be the proposition that “ x is an equilateral polygon” and $EA(x)$ be the proposition that “ x has all equal interior angles,” and $HP(x)$ be the statement that x has two parallel sides.
- (a) What I would like to do is write down a definition of a square, rectangle and rhombus using these truth valued statements so I make a statement like

$HP(x)$ if and only if $Rh(x)$

$(EA(x) \text{ and } HP(x))$ if and only if $R(x)$

$(EP(x) \text{ and } EA(x) \text{ and } HP(x))$ if and only if $S(x)$

but I found that I missed the mark because when I tried I could draw examples of figures where the statements were false. Give one example of a shape (which will be the x in the statement) where all three of the statements are false.

- (b) Modify the statements that I made above so that they do reflect the definitions of rhombus, rectangle and square.
- (c) Which of the following are always true: $Rh(x) \text{ implies } R(x)$, $Rh(x) \text{ implies } S(x)$, $R(x) \text{ implies } Rh(x)$, $R(x) \text{ implies } S(x)$, $S(x) \text{ implies } R(x)$, $S(x) \text{ implies } Rh(x)$?
- (2) Prove or disprove the statement: “If x is a real number such that $|x + 1| + |x - 1| \leq 1$, then $|x^2 - 1| \leq 1$.”
- (3) Prove or disprove the statement: “Let $a, b, c \in \mathbb{Z}$, if ab, ac, bc are all even, a, b, c are even.”