ASSIGNMENT #4

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

(1) Consider the set of complex numbers $\{1, i, -1, -i, \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\}$.
   (a) Create an 8×8 multiplication table of the elements in this set.
   (b) Which elements z in the set have the property that $z^0, z^1, z^2, \ldots, z^7$ are all distinct?
   (c) Write each of the elements in the form $e^{\theta i}$.
   (d) Which subsets of these elements S have the property that $g_1 \cdot g_2$ is an element of S for all g_1 and g_2 in S? If you give a list of these subsets, explain how you know that you have produced an exhaustive list?

(2) Prove or disprove the statement: "If x is a real number such that $|x+1| + |x-1| \le 2$, then $|x^2 - 1| \le 1$."

(3) A number which can be represented as the number of dots in a triangular array is called a 'triangular' number

A number which can be represented in a pentagonal array is called pentagonal number.
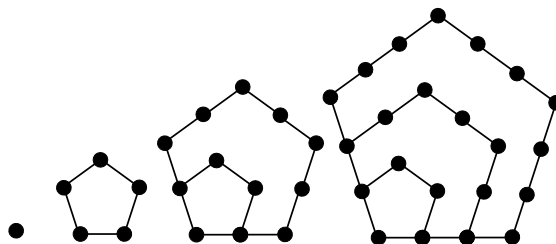
Which numbers are triangular? Which number are pentagonal? Are there any triangular numbers which are also pentagonal numbers? Why or why not?