ASSIGNMENT #5

DATE: NOVEMBER 14, 2018 DUE: NOVEMBER 27, 2018

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

(1) Prove (by induction or by telescoping sums or by some other means) that for $n \geq 0$ and $x \neq 1$,
$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x} \ .$$
(2) Provide three proofs of the following identity for $n \geq 1$:
$$1 + 2 \cdot 5^1 + 3 \cdot 5^2 + \cdots + n \cdot 5^{n-1} = \frac{1 - (n+1)5^n + n5^{n+1}}{4^2}$$
   (a) by telescoping sums
   (b) by induction
   (c) by taking the derivative of the identity in problem (1) and setting $x = 5$
(3) Show that if $\phi = \frac{1+\sqrt{5}}{2}$, then $\phi^{2n} = F_{2n-2} + \phi F_{2n-1}$ for $n \geq 1$ where F_n is the Fibonnacci sequence where $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Find and conjecture a similar formula involving $\overline{\phi} = \frac{1-\sqrt{5}}{2}$.
(4) Prove that if $a_1, a_2, \ldots, a_n \geq 1$, then
$$2^{n-1}(a_1 a_2 \cdots a_n + 1) \geq (1 + a_1)(1 + a_2) \cdots (1 + a_n) \ .$$
(5) Prove that $7^{2n} - 48n - 1$ is divisible by 2304 for every natural number n (remember that to say a number m is divisible by 2304 means that $m = 2304k$ where k is an integer).