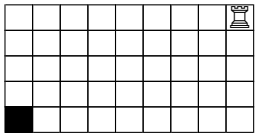
ASSIGNMENT #6

DATE: JANUARY 6, 2019 DUE: JANUARY 20, 2019

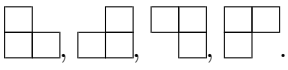
Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

(1) A rook chess piece is placed on the upper right hand corner of an $m \times n$ grid (see for instance the example of a 5×9 grid below). In the lower left hand corner of the grid there is a death trap.

death trap → 

Players alternate in turns by moving the chess piece either some (greater than 0) number of spaces to the left or some number of spaces down always closer to the death trap. The person who moves the rook piece onto the space with the death trap loses.

- For which values of m and n does the first player have a winning strategy in the game?
- For which values of m and n does the second player have a winning strategy in the game?
- Generalize the game to three dimensional chess board with $m \times n \times k$ pieces and determine when the first player has a winning strategy.

(2) Show that every square grid of size $2^n \times 2^n$ minus the upper right hand corner can be tiled using shapes of the form  . Use the argument that you describe (that this works for all n) to construct the tiling for an 8×8 square grid minus the upper right hand corner.

(3) Show that 3 divides $4^n - 1$ and show that if $k_n = 1 + 4 + 4^2 + \cdots + 4^{n-1}$ then $3k_n = 4^n - 1$.