ASSIGNMENT #7

DATE: JANUARY 22, 2019 DUE: FEBRUARY 3, 2019

Your assignment should include complete sentences and explanations and not just a few equations or numbers. A solution will not receive full credit unless you explain what your answer represents and where it came from. You may discuss the homework with other students in the class, but please write your own solutions.

(1) Alice was born in 2000 and has a birthday on February 29. Find the day of the week her birthday occurred on in 2000, 2004, 2008, 2012, ..., 2016. Explain this pattern and find a way of determining the day of the week in any year up to 2100.

What goes wrong in 2100? Explain how to find the day of the week in any year even after 2100.
Note: In the Gregorian calendar, years that are divisible by 100, but not by 400, do not contain a leap day.

(2) Bob was born in 2000 and has a birthday on February 28. Find the day of the week his birthday occurred on in 2000 through 2018. Explain this pattern and find a way of determining the day of the week in any year up to 2100.

Figure out a way to determine the day of the week in any year even after 2100.

(3) Prove by providing an argument or disprove by finding a counterexample to the following statements. a, b are positive integers and r, s are integers.
  (a) if $r \equiv s\ (mod\ a)$, then $s \equiv r\ (mod\ a)$.
  (b) if $r \equiv s\ (mod\ a)$, then $r \equiv s\ (mod\ ab)$.
  (c) if $r \equiv s\ (mod\ ab)$, then $r \equiv s\ (mod\ a)$.
  (d) if $r \equiv s\ (mod\ a)$, then $kr \equiv ks\ (mod\ a)$.
  (e) if $kr \equiv ks\ (mod\ a)$, then $r \equiv s\ (mod\ a)$.
  (f) if $rs + ab = 1$ then $rs \equiv 1\ (mod\ a)$.
  (g) there exists an $x \in \{0, 1, 2, \ldots, a-1\}$ such that $s \equiv x\ (mod\ a)$.
  (h) if $s \equiv 0\ (mod\ a)$ then $a\ divides\ s$.
  (i) if $s \equiv 0\ (mod\ a)$ then $s\ divides\ a$.
  (j) if $b \equiv 0\ (mod\ a)$ and $a \equiv 0\ (mod\ b)$, then $a = b$.