A PROOF BY CONTRADICTION

We started with five rules for ordering of the real numbers.

(1) If $x \in \mathbb{R}$, then $x > 0$, $x = 0$ or $x < 0$ and exactly one of these is true.
(2) If $x > y$, then $-x < -y$.
(3) If $x > y$ and $c \in \mathbb{R}$, then $x + c > y + c$.
(4) If $x > 0$ and $y > 0$, then $xy > 0$.
(5) If $x > y$ and $y > z$, then $x > z$.

Theorem 1. *Let x, u and v be real numbers. If $x > 0$ and $u > v$, then $xu > xv$.*

Proof. We start by assuming the opposite of 'If $x > 0$ and $u > v$, then $xu > xv$.' A statement of the form 'if P, then Q' is false if P is true, and Q is false. Hence, we assume $x > 0$ and $u > v$ and $xu \leq xv$ (in this case P is the statement '$x > 0$ and $u > v$' and Q is the statement $xu > xv$).

By rule (3), $u - v > v - v$ so $u - v > 0$. By rule (4), $x(u - v) > 0$ and so $xu - xv > 0$.

But our assumption was that $xu \leq xv$ and so either $xu < xv$ (and so $xu - xv < 0$ by rule (3)) or $xu = xv$ (and so $xu - xv = 0$). But by rule (1), it is impossible that $xu - xv > 0$ and either $xu - xv < 0$ or $xu - xv = 0$. This is a contradiction. Therefore 'If $x > 0$ and $u > v$, then $xu > xv$' is true. □

You should try to write this as a direct proof and to do this by assuming that '$x > 0$ and $u > v$' and showing that $xu > xv$.