A PROOF THAT $\sqrt{6}/3$ IS IRRATIONAL

MIKE ZABROCKI

Theorem 1. $\sqrt{6}/3$ *is irrational*

Note: the technique of proving that $\sqrt{a}$ is irrational is 'assume that rational' and arrive at a contradiction. The contradiction is usually to show that the rational number that it is equal to cannot be a fraction with no common factors in the numerator and denominator (which we should always be able to do).

Proof. Assume that $\sqrt{6}/3 = \frac{r}{s}$ where r and s are integers and $s \neq 0$. Moreover, we can assume that r and s have no common factors.

If we square this equation then $\frac{r^2}{s^2} = \frac{6}{9} = \frac{2}{3}$, and so $3r^2 = 2s^2$. Since the product of odd numbers is odd, if r is odd, then $3r^2$ is odd, but since $2s^2$ is even and $2s^2 = 3r^2$, it must be that r is even. Hence $r = 2k$ for some integer k and $3r^2 = 12k^2 = 2s^2$ so $s^2 = 6k^2$.

Now s cannot be odd since $s^2 = 6k^2$ and $6k^2$ is even so s must be even. But our assumption was that r and s have no common factors, but we deduced that they were both even, hence this is a contradiction. □