SOME SUMS

Use the technique of telescoping sums to prove the following

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$
$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$
$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(4n^2-1)}{3}$$
$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$
$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$
$$1 \cdot 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5 + 3 \cdot 4 \cdot 5 \cdot 6 + \cdots + n(n+1)(n+2)(n+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$
$$1^2 \cdot 3 + 2^2 \cdot 5 + 3^2 \cdot 7 + \cdots + n^2(2n+1) = \frac{n(n+1)(3n^2+5n+1)}{6}$$
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$
$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(4n^2-1)}{3}$$
$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2-1)$$
$$1! \cdot 1 + 2! \cdot 2 + 3! \cdot 3 + \cdots + n! \cdot n = (n+1)! - 1$$
$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}$$
$$\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \cdots + \frac{n}{3^n} = \frac{3}{4}\left(1 - \frac{1}{3^n} + n\frac{1}{3^{n+1}} - n\frac{1}{3^n}\right)$$

Conjecture and prove a product formula for the following expressions:

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n+2)$$
$$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \cdots + (2n+1)(2n+3)$$
$$\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{n(n+2)}$$
$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n+1)(2n+3)}$$