HOMEWORK #6

DATE: MARCH 18, 2020 : DUE: APRIL 1, 2020

In class we have been talking about relations and we have defined the the notions of reflexive, symmetric and transitive. A relation R on a set D is called *anti-reflexive* if $(x, x) \notin R$ for any $x \in D$. A relation R is called *anti-symmetric* if $(x, y) \in R$ with $x \neq y$, then $(y, x) \notin R$. A relation is called *circular* if $(x, y) \in R$ and $(y, z) \in R$, then $(z, x) \in R$.

(1) Prove that it is not possible to have a relation on a non-empty set D which is both reflexive and anti-reflexive. Note that if D is empty, it is possible to have a relation which is both reflexive and anti-reflexive so make sure that your explanation reflects this.

(2) Choose $D = \{1, 2, 3, 4\}$ to be the domain of a relation. For each of the 8 combinations

$$\begin{aligned}\{(A, B, C) : &A \in \{\text{anti-reflexive}, \text{not anti-reflexive}\},\\ &B \in \{\text{anti-symmetric}, \text{not anti-symmetric}\},\\ &C \in \{\text{circular}, \text{not circular}\}\}\end{aligned}$$

give at least one example of a relation which satisfies exactly those possibilities. For some combinations of properties, it may be that no such relation exists. In these cases, prove that it is impossible by providing an explanation why.

(3) A *partially ordered set* (sometimes called a *poset*) is a set D together with a relation on D which is reflexive, anti-symmetric and transitive. Prove that the positive integers together with the relation

$$\{(a, b) : a, b \in \mathbb{Z}, a \text{ divides } b\}$$

is a partially ordered set.

(4) Prove that the relation on the complex numbers defined by

$$R = \{(x, y) : x, y \in \mathbb{C}, x = ay \text{ for some } a \in \mathbb{R}, a > 0\}$$

is an equivalence relation. Give a description of the equivalence classes of the relation.