

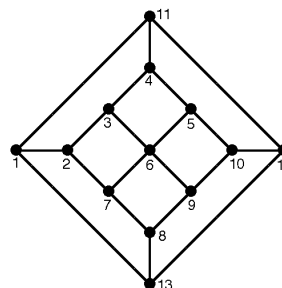
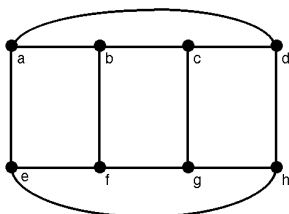
HOMEWORK #1 - MATH 3260

ASSIGNED: JANUARAY 20, 2003

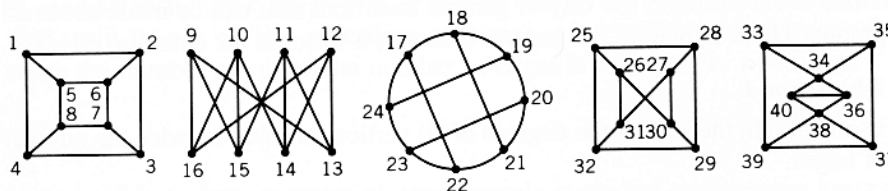
DUE: JANUARY 29, 2002 AT 2:30PM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3) however you must register your group by the January 27, 2002 either by e-mail to zabrocki@mathstat.yorku.ca or in class with the sign-up sheet. You need only hand in one assignment per group, and write all names at the top.

- (1) Explain clearly what is the largest possible number of vertices in a graph with 19 edges and all vertices of degree at least 3. Explain why this is the maximum value.
- (2) Determine whether the following graphs are bipartite. If so, give the partition of the vertices into two sets.



- (3) Which pairs of the following set of graphs are isomorphic.



Give explicitly the isomorphisms between the pairs of graphs.

- (4) The **line graph** $L(G)$ of a simple graph G is the graph whose vertices are in one-to-one correspondence with the edges of G , two vertices of $L(G)$ being adjacent if and only if the corresponding edges of G are adjacent.
 - (a) Show that $L(K_3)$ and $L(K_{1,3})$ are isomorphic.
 - (b) Show that the line graph of the tetrahedron graph is isomorphic to the octahedron graph.
 - (c) Find an expression for the number of edges of $L(G)$ in terms of the degrees of the vertices of G .
 - (d) Prove that if G is regular of degree k , then $L(G)$ is regular of degree $2k - 2$.
- (5) The **complete tripartite graph** $K_{r,s,t}$ consists of three sets of vertices (of sizes r, s and t), with an edge joining two vertices if and only if they lie in different sets. Draw the graphs $K_{2,2,2}$, $K_{3,3,2}$ and find the number of edges of $K_{3,4,5}$.