

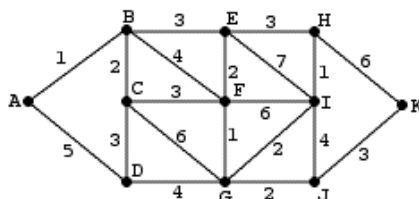
## HOMEWORK #2 - MATH 3260

ASSIGNED: JANUARAY 30, 2003

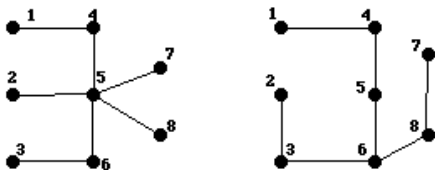
DUE: FEBRUARY 12, 2002 AT 2:30PM

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers. You may work in groups (maximum 3) however you must register your group by the January 27, 2002 either by e-mail to [zabrocki@mathstat.yorku.ca](mailto:zabrocki@mathstat.yorku.ca) or in class with the sign-up sheet. You need only hand in one assignment per group, and write all names at the top.

- (1) (a) Give by listing the sequence of vertices 4 Hamiltonian cycles in  $K_9$  no two of which have an edge in common.
- (b) What is the maximum number of edge disjoint Hamiltonian cycles in  $K_{2k+1}$ ?
- (2) Find the shortest path from  $A$  to each of the other vertices in the weighted graph of the figure below. Draw a spanning tree rooted at  $A$  with smallest weight. Are there others?



- (3) Let  $T_1$  and  $T_2$  be spanning trees of a connected graph  $G$ .
  - (a) If  $e$  is any edge of  $T_1$ , show that there exists an edge  $f$  of  $T_2$  such that the graph with edge set equal to  $E(T_1) - \{e\} \cup \{f\}$  (obtained from  $T_1$  by replacing  $e$  by  $f$ ) is also a spanning tree.
  - (b) Transform the graph on the left to the one on the right by a sequence of trees each of which differs from the next by a single edge. Explain why this can be done for any two trees with the same vertex set.



- (4) How many spanning trees does  $W_n$  have for  $n \geq 4$ ?
- (5) Prove that any tree which is semi-Hamiltonian is isomorphic to  $P_n$ .