

MIDTERM - TAKE HOME - MATH 4160

ASSIGNED: OCTOBER 23, 2014

DUE: OCTOBER 28, 2014 AT 2:30PM

Write your solutions neatly and clearly. Provide full explanations and justify all of your answers. DO NOT DISCUSS THESE PROBLEMS WITH OTHERS. You must do this work alone and I will ask you to sign the statement below which states that you have not discussed these problems with others or received help on these problems (when you hand the paper to me). Note that in certain circumstances I am giving you the answer, and it is your job *explain* it. This means that you should write grammatically correct sentences, tell me why two things are equal, and make your calculations clear and easy to follow.

If you have any questions about the problems you may e-mail me at zabrocki@mathstat.yorku.ca.

1 (4 pts)	.
2 (6 pts)	
3 (4 pts)	
4 (4 pts)	
total(18 pts)	

- (1) (4 pts) Define the sequence $d_0 = 1$ and $d_1 = 1$ and $d_n = -2d_{n-1} + \sum_{i=0}^{n-1} 2^i d_{n-i-1}$ for $n \geq 2$. Compute the values of d_2 through d_5 . Find the generating function of this sequence

$$D(x) = \sum_{n \geq 0} d_n x^n.$$

Hint: If you did this right then you should be able to use the computer to show that $d_{10} = 2,929$.

- (2) (6 pts) For the following identity give a combinatorial proof by describing a set that is counted by the left hand side of the equality and a set that is counted by the right hand side of the equality and explaining why these two sets are the same size.

$$\binom{3n+1}{n} = \sum_{k=0}^n \binom{n+k}{n} \binom{2n-k}{n}.$$

- (3) (4 pts) Give an expression for the generating function for the number integer solutions to

$$x_1 + x_2 + 2x_3 + 3x_4 + 3x_5 + 2x_6 + x_7 + x_8 = n$$

with $x_i \geq 0$ and $x_2 + 3x_4 + 2x_6 + x_8$ is odd.

Hint: There are 26,566 solutions when $n = 20$.

- (4) (4 pts) Calculate the number of ways of making change for \$18.55 using nickles, dimes, quarters, loonies and toonies where at most \$1 is made up of nickles and dimes.

Hint: the answer is 1910

Bonus (4 pts): Calculate the generating function for the number of ways of making change if an even number of coins are used.

WHEN YOU SUBMIT THIS EXAM please sign the following statement and fill out the information below.

I attest that I have completed this exam myself without help from anyone else and I have not discussed the problems on this exam with other students in the class.

This exam is open book, open notes, and other sources, but I expect you to not ask other people how to complete the assignment. Everyone should list books and websites that you consulted below. Your solutions should refer to precise pages in the notes and reference that I can consult. If you cannot sign the above statement truthfully, I would prefer if you just explain to me the situation rather than perjure yourself. Please detail below the sources you consulted, the that you have obtained on this exam or who you have discussed these problems with: