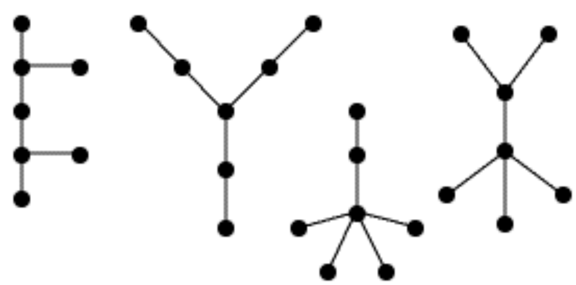
HOMEWORK #4 - MATH 4160

ASSIGNED: FRIDAY NOVEMBER 17, 2017 DUE: THURSDAY, NOVEMBER 30, 2017

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers.

(1) For each of the following four trees (they are called trees because there are no loops and only branches), how many different ways are there of coloring the vertices with k colors so that no two adjacent vertices are colored with the same color?



(2) For each of the trees in the first problem, how many different ways are there of coloring the vertices with k colors (no restriction on which colors can be used) where two colorings of the graph are considered different if there is no rearrangement of the vertices so that the colorings look the same?
(3) For each of the trees in the first problem, how many different ways are there of coloring the vertices with k colors such that adjacent vertices are colored with different colors and so that two colorings of the graph are considered different if there is no rearrangement of the vertices so that they look the same?

(4) The Snub Cube (`http://mathworld.wolfram.com/SnubCube.html`, `https://en.wikipedia.org/wiki/Snub_cube`) has 6 square faces and 32 triangular faces. Allowing for only rotations of the object (no mirror reflections), how many elements are there in the group of symmetries of this object? Explain clearly your count. List the elements of the group as a permutation of the 32 triangular faces.
(5) How many different (under motions of the symmetry group) ways are there of coloring the 32 triangular faces of the snub cube with black and white?
(6) How many ways are there of coloring the triangular faces of the snub cube with black and white using 16 black faces and 16 white faces?

(7) Consider a partial order on piles of 2 black stones and 2 white stones. Say that one configuration of pile of stones, A, is smaller than another, B, if you can join piles of A together to get those of B. The smallest configuration under this order is $\circ|\circ|\bullet|\bullet$ while the largest is $\circ\circ\bullet\bullet$. Draw the Hasse diagram for these configurations with this partial order.

(8) Compute the values of the Möbius function of the form $\mu(\circ|\circ|\bullet|\bullet, A)$ for this partial order for all configurations A.