

(1) **Proposition:**

$$\text{For } k \geq 0 \text{ and } n \geq 0, 1_{(k)} + 2_{(k)} + \dots + n_{(k)} = \frac{(n+1)_{k+1}}{k+1}$$

Proof: Let $1_{(k)} + 2_{(k)} + \dots + n_{(k)} = a_1 + a_2 + \dots + a_n = b_n$

$$\text{Since } b_n = \frac{(n+1)_{k+1}}{k+1}$$

$$\text{then } b_i = \frac{(i+1)_{k+1}}{k+1} \text{ and } b_{i-1} = \frac{(i)_{k+1}}{k+1}$$

$$\begin{aligned} b_i - b_{i-1} &= \frac{(i+1)_{k+1}}{k+1} - \frac{(i)_{k+1}}{k+1} \\ &= \frac{(i+1)(i)(i-1)\dots(i-k+1) - (i)(i-1)\dots(i-k+1)}{k+1} \\ &= \frac{(i)(i-1)\dots(i-k+1)}{k+1} [(i+1) - (i-k)] \\ &= \frac{(i)(i-1)\dots(i-k+1)}{k+1} (k+1) \\ &= (i)(i-1)\dots(i-k+1) \\ &= i_{(k)} = a_i \end{aligned}$$

$$b_0 = (0+1)_{k+1} = (1)_{k+1} = 1 * 0 * \dots * (1-k+1) = 0$$

Therefore, by telescoping sums,

$$a_1 + a_2 + \dots + a_n = 1_{(k)} + 2_{(k)} + \dots + n_{(k)} = b_n = \frac{(n+1)_{k+1}}{k+1}$$

good

1. States the problem
2. defines notation
3. complete calculations
4. states the conclusion
5. states why conclusion is true
6. uses full sentences

This is missing a few periods at the end of the sentence but otherwise is good.

1. Consider $\sum_{i=1}^n (i)_k = \frac{(n+1)_{k+1}}{k+1}$ as a telescoping sum, with $b_i = \frac{(i+1)_{k+1}}{k+1}$

$$\begin{aligned} b_i - b_{i-1} &= \frac{(i+1)_{k+1}}{k+1} - \frac{(i)_{k+1}}{k+1} \\ &= \frac{(i+1)_{k+1}}{k+1} - \frac{(i)_{k+1}}{k+1} \\ &= \frac{(i+1)(i)(i-1)\dots(i-k+1)}{k+1} - \frac{i(i-1)\dots(i-k+1)(i-k)}{k+1} \end{aligned}$$

Taking the factor of $\frac{(i)(i-1)\dots(i-k+1)}{k+1}$

$$\begin{aligned} &= (i+1 - (i-k)) \frac{(i)(i-1)\dots(i-k+1)}{k+1} \\ &= (k+1) \frac{(i)(i-1)\dots(i-k+1)}{k+1} \\ &= (i+1)_k \end{aligned}$$

This is the next term in the sequence

$$b_0 = 0$$

as it is the sum of zero

good. Except you should write your conclusion.
What does this show?

this solution has a lot of the same elements. It is mostly good, but it is missing the statement of the problem and the conclusion

2 (2) Please write the problem.

Let $n=1$,

$$\text{left} = 1, \text{right} = (1 \times 4)/4 = 1, \quad \text{what is "left" what is "right"}$$

left=right, it is true. Don't use undefined notation.

Let $n=k$, assume the statement is true,

Your left hand side depends on n
and your right hand side depends on k
so there is a problem. Don't use summation notation,
it isn't helping to make it clearer here since it is wrong

$$\sum_{k=1}^n k^3 = \frac{k^2(k+1)^2}{4}$$

$$\text{Then to show } 1^3 + 2^3 + 3^3 + \dots + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3,$$

$$\text{right} = k^2(k+1)^2 + 4(k+1)^3 \cdot \frac{1}{4}$$

$$= (k+1)^2(k+4k+4)^2/2$$

$$= (k+1)^2(k+2)^2/4$$

why are some of the k 's in the equation
and others not in the equation? (some are italics, others
are in roman)

Thus, left=right, this statement is true.

why does left=right imply that the statement is true?

this one leaves a lot to be desired. No statement of the problem,
no sentences, mistakes in the calculation.

(Q2)

$$p(\text{three of a kind}) = \left(\binom{11}{1} \binom{4}{3} \binom{46}{2} + \binom{2}{1} \binom{3}{3} \binom{47}{2} \right) / \binom{50}{5}$$

there are two situation in these case, the first situation is choose 1 card from 11 cards of different rank but all are spade, there are 2 cards missing so $13-2=11$ cards, then accoeding to the rank of card that we choose, choose 3 cards from

This one doesn't state the problem and the answer is a probability rather than answering the question which was to give the number hands. The "explanation" is a bit of a string of words.

subset, so $\mathfrak{S}(n,n)=1$

8 b

(8b) This is fine that you followed my explanation and it is appropriate that you cited it. It is inappropriate to use the text word for word without putting quote marks around it. Be extremely careful. If you hadn't been so explicit in your parenthetical remark, I would have accused you of academic dishonesty.

(I did the part b by looking at your note in 2012, I use you proof here because I can't have a better answer on this question, I can understand your proof peocedure but I just can't have a better proof by other method, so it is ok if you don't give me marks for this part)

let $[n] := \{1, 2, \dots, n\}$ The set partitions of $[n]$ into k parts can be divided into two sets, the first have n in a part by itself, have n in a part by itself, the second have n in a part with other values from $[n-1]$: $\{1, 2, \dots, n-1\}$

Just a reminder: There are rules about using text from elsewhere. You can borrow ideas (just indicate from where you got them), but don't borrow text (or if you do, put quotes around it). Universities draws a clear line about what is considered plagiarism.

base case n=1

$$\text{LHS} = (x)^{(1)} = x$$

$$\text{RHS} = s'(1,1)x = 1x = x = \text{LHS}$$

So true for n=1

Assume that the equation is true for n=m, m is a fixed constant, then

use full sentences. Spell correctly.

"So true for n=1" = "So the equation holds when n=1"

or "So the equation is true for n=1."

True is not a verb, it is an adjective.

$$(x)^{(m)} = x(x+1)\dots(x+m-1) = \sum_{k=1}^m s'(m, k)x^k = s'(m, 1)x + s'(m, 2)x^2 + \dots + s'(m, m)x^m$$

let x=m+1, then **What is with the non spaces?**
Why is x=m+1?

$$(x)^{(m+1)} = x(x+1)\dots(x+m-1)(x+m) = (x+m)\left(\sum_{k=1}^m s'(m, k)x^k\right)$$

$$= s'(m, 1)x(x+m) + s'(m, 2)x^2(x+m) + \dots + x^m(x+m) = \sum_{k=1}^{m+1} s'(m+1, k)x^k$$

so true for n=m+1

by induction, the equation is true as proved.

You are missing the calculation that shows why the right hand side follows from the left hand side of this equality. something different happens for the coefficient of x^{m+1} and x^0 than for all the other coefficients.

☞ question ☞

I had one more example, the structure of the proof is right, but the details are missing (and there are a few mistakes). This one also didn't state the problem or give a conclusion except that "the equation is true as proved."