HOMEWORK #4 - MATH 4160

ASSIGNED: FRIDAY NOVEMBER 21, 2019 DUE: THURSDAY, DECEMBER 5, 2019

Write your homework solutions neatly and clearly. Provide full explanations and justify all of your answers.

(1) A simple graph consists of a set of vertices V and a set of edges E where an edge $e \in E$ is a subset $e \subseteq V$ of size 2 (edges must have distinct vertices so you can't have loops of the form $\{i, i\}$). Two graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ are said to be isomorphic if $|V_1| = |V_2|$ and there exists a bijection $\phi : V_1 \to V_2$ such that $\{a, b\} \in E_1$ if and only if $\{\phi(a), \phi(b)\} \in E_2$. The complete graph is denoted K_n is the graph $(\{1, 2, \ldots, n\}, \{\{a, b\} : 1 \leq a < b \leq n\})$. Compute the number of non-isomorphic graphs with $1, 2, 3, 4$ and 5 vertices by counting the number of ways of coloring the edges of the complete graph K_r for $1 \leq r \leq 5$ with colors black and white (then a representative graph is one with the edge set consists of the just the black edges of the coloring).
Note the for $r = 1$ and 2, you need not set this up as an application of Burnside's Lemma/Pólya's Theorem, but for $r = 3$ you can verify that Burnside's Lemma/Pólya's Theorem gives the same answer that you compute by hand.
(2) Use the principle of inclusion-exclusion to compute the number of connected graphs with size $1, 2, 3, 4$ and 5 vertices.

(3) The Pentakis Dodecahedron (you may want to build your own) has 60 triangular faces, 90 edges and 32 vertices. Allowing for only rotations of the object (no mirror reflections), how many elements are there in the group of symmetries of this object? Explain clearly your count. List the elements of the group or at least categorize them by their cycle structure as a permutation of the 32 vertices.
(4) How many different (under motions of the symmetry group of rotations) ways are there of coloring the 32 vertices of the Pentakis Dodecahedron black and white?
(5) How many ways are there of coloring the 32 vertices of the Pentakis Dodecahedron with black and white using 16 black vertices and 16 white vertices?

(6) Consider the partial order on permutations of $\{1, 2, 3, 4\}$. For π a permutation of $\{1, 2, 3, 4\}$, define $D(\pi) = \{i : 1 \leq i \leq 3 \text{ and } \pi(i) > \pi(i+1)\}$. For two permutations π, τ of $\{1, 2, 3, 4\}$, say that $\pi <_* \tau$ if $D(\pi) \subsetneq D(\tau)$. Draw the Hasse diagram of this order $\leq_*$.
(7) Compute the values of the Möbius function of the form $\mu(1234, \pi)$ for this partial order for all permutations π of $\{1, 2, 3, 4\}$.