# Math 4160: Combinatorial Mathematics - Fall 2019

## Contact information:

Mike Zabrocki
Course will take place WC 118 - TR 2:30-3:45pm
Office: TEL/DB 2026
email: zabrocki@mathstat.yorku.ca
office hours: Monday 11-1pm and Thursday 1-2pm or by appointment

## Course description:

Topics from algebra of sets, permutations, combinations, occupancy problems, partitions of integers, generating functions, combinatorial identities, recurrence relations, inclusion-exclusion principle, Pólya's theory of counting, permanents, systems of distinct representatives, Latin rectangles, block designs, finite projective planes, Steiner triple systems. Prerequisites: SC/MATH 2022 3.00 or SC/MATH 2222 3.00; six credits from 3000-level mathematics courses without second digit 5; or permission of the course coordinator.

The main goal of this course will be to develop four common tools of combinatorics that are used to enumerate finite sets: basic counting principles, bijective arguments, Pólya Theory and generating functions. We will apply these tools to examine specific examples of combinatorial objects. The emphasis of assignments will be on problem solving and development of combinatorial techniques.

## Grade:

The grade for this course will mostly be based on homework assignments (55%). You may work together on these assignments, but I want you to hand in your own work (YOU must work on these problems and they will be compositions so I expect your answers to be original even if you worked with someone). There will be a midterm (20%) and a final (25%) and, although these assignments are take home, you will be expected to work on those problems on your own. You will be able to consult with me on the midterm and final. On the take home assignments I expect original work.

# Course references:

- There is no required textbook for this course, but the material that we will cover can be found in many references. If you feel that you want a textbook, a good one that covers this is How to Count: An Introduction to Combinatorics (https://www.crcpress.com/How-to-Count-An-Introduction-to-Combinatorics-Second-Edition/Allenby-Slomson/p/book/9781420082609).
- For the topic of generating functions I wrote a short book that includes videos that summarize the text.
  An Introduction to Ordinary Generating Functions (http://garsia.math.yorku.ca/~zabrocki/MMM1/)
- I have taught this course four times before (2003 (http://garsia.math.yorku.ca/~zabrocki/math4160w03/), 2012 (http://garsia.math.yorku.ca/~zabrocki/math4160f12/), 2014 (http://garsia.math.yorku.ca/~zabrocki/math4160f14/), 2017 (http://garsia.math.yorku.ca/~zabrocki/math4160f17/)). The other times I taught the course I took detailed notes and updated them as I went along:
  - Chapter 1: Finite Sums (10 pages) (notes/ch1_finite_sums.pdf)
  - Chapter 2: Counting using Recursion (8 pages) (notes/ch2_counting_recursion.pdf)
  - Chapter 3: Counting using Bijections (8 pages) (notes/ch3_counting_bijections.pdf)
  - Chapter 4: Generating Functions (32 pages) (notes/ch4_generating_functions.pdf)
  - Chapter 5: Exponential Generating Functions (8 pages) (notes/ch5_exponential_gfs.pdf)
  - Chapter 6: Pólya Enumeration (43 pages) (notes/ch6_polya.pdf)
  - Chapter 7: Mobius Inversion (16 pages) (notes/ch7_mobius_inversion.pdf)

  I will probably not have the time to write detailed notes this year, but I might try to improve what I have into a better form if we cover similar topics.
- Most assignments in this course are more about writing than being able to calculate. You will need to LaTeX your assignments (latex.html).
- When I do examples in class or in the notes I will sometimes augment a calculation with a calculation in the computer algebra system Sage (http://www.sagemath.org/) (e.g. to take a coefficient in a series). Sage is free to download or you can access Sage online through the sage cell server (https://sagecell.sagemath.org/) or by opening a Cocalc (https://cocalc.com/) account. You are welcome to use programs you are more familiar with (e.g. Maple, Matlab or Mathematica) to do the same calculations or there are specific online tools that can do these calculations.
- Other material for this course, announcements and grades will be on the course moodle (http://moodle.info.yorku.ca/).
- Homework assignments:
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# Schedule:

| Date | Topic | Remarks |
|---|---|---|
| Sept 5 | Compute $1^k + 2^k + \cdots + n^k$ | telescoping sums (https://www.youtube.com/watch?v=H6MmDRtuiNw), notes (files/telescoping.pdf) |
| Sept 10 | finish using Stirling numbers to compute $\sum_{i=1}^{n} i^k$ | |
| Sept 12 | sets and multisets | Watch video: addition/multiplication principle (https://www.youtube.com/watch?v=gipHUb9UeeU) |
| Sept 17 | Set partitions, permutations | First assignment (files/ass1.pdf) - due Oct 3 |
| Sept 19 | counting poker hands | |
| Sept 24 | | |
| Sept 26 | | |
| Oct 1 | | |
| Oct 3 | | |
| Oct 8 | | |
| Oct 10 | | |

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| Oct 15 & 17 | Reading Week | |
| Oct 22 | | |
| Oct 24 | | |
| Oct 29 | | |
| Oct 31 | | |
| Nov 5 | | |
| Nov 7 | | |
| Nov 12 | | |
| Nov 14 | | |
| Nov 19 | | |
| Nov 21 | | |
| Nov 26 | | |
| Nov 28 | | |
| Dec 3 | | |

# Announcements:

(Sept. 2, 2019) The first day we are going to look at how combinatorics can be used to compute sums of the form $1^r + 2^r + \ldots + n^r$. We will just do a bit of experimenting and conjecturing today, but then I want you to go home, watch some videos and do some homework and we will get to the punchline in the second class.
Please watch the following videos:

- The addition and multiplication principles (https://www.youtube.com/watch?v=gipHUb9UeeU)
- Telescoping sums (https://www.youtube.com/watch?v=H6MmDRtuiNw)
  and make sure you read the page about writing your homework in LaTeX (latex.html)

Then I would like you to to prove the following results using telescoping sums:

- $1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
- $1^4 + 2^4 + \ldots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n \cdot (n+2) =$

Start typing up those in LaTeX. You don't have to hand it in next time, but you will hand it in when I give you your first homework assignment.

(Sept 17, 2019) I've posted the first assignment (files/ass1.pdf) that will be due October 3, 2019.