SOME INCLUSION-EXCLUSION PROBLEMS

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The following enumeration problems can be solved using the principle of inclusion-exclusion. Set up the solution to these questions by naming sets $A_1, A_2, \cdots, A_k$ such that either the set or the complement of the set is counted by the inclusion-exclusion formula.

(1) How many n-digit decimal sequences (using the digits $0-9$) are there in which the digits $1, 2$ and 3 all appear?
(2) How many ways are there of rolling a sided die 10 times in a sequence such that all 6 faces appear?
(3) How many positive integers ≤ 420 are relatively prime to $420 = 5 \cdot 2^2 \cdot 7 \cdot 3$?
(4) How many arrangements of 52 letters, 2 As, 2 Bs, 2 Cs, etc. with no pair of consecutive letters the same?
(5) How many ways are there of dealing a 13 card hand with at least one void in a suit?
(6) How many 13 card hands have at least one type of face card (J, Q, K, A)?