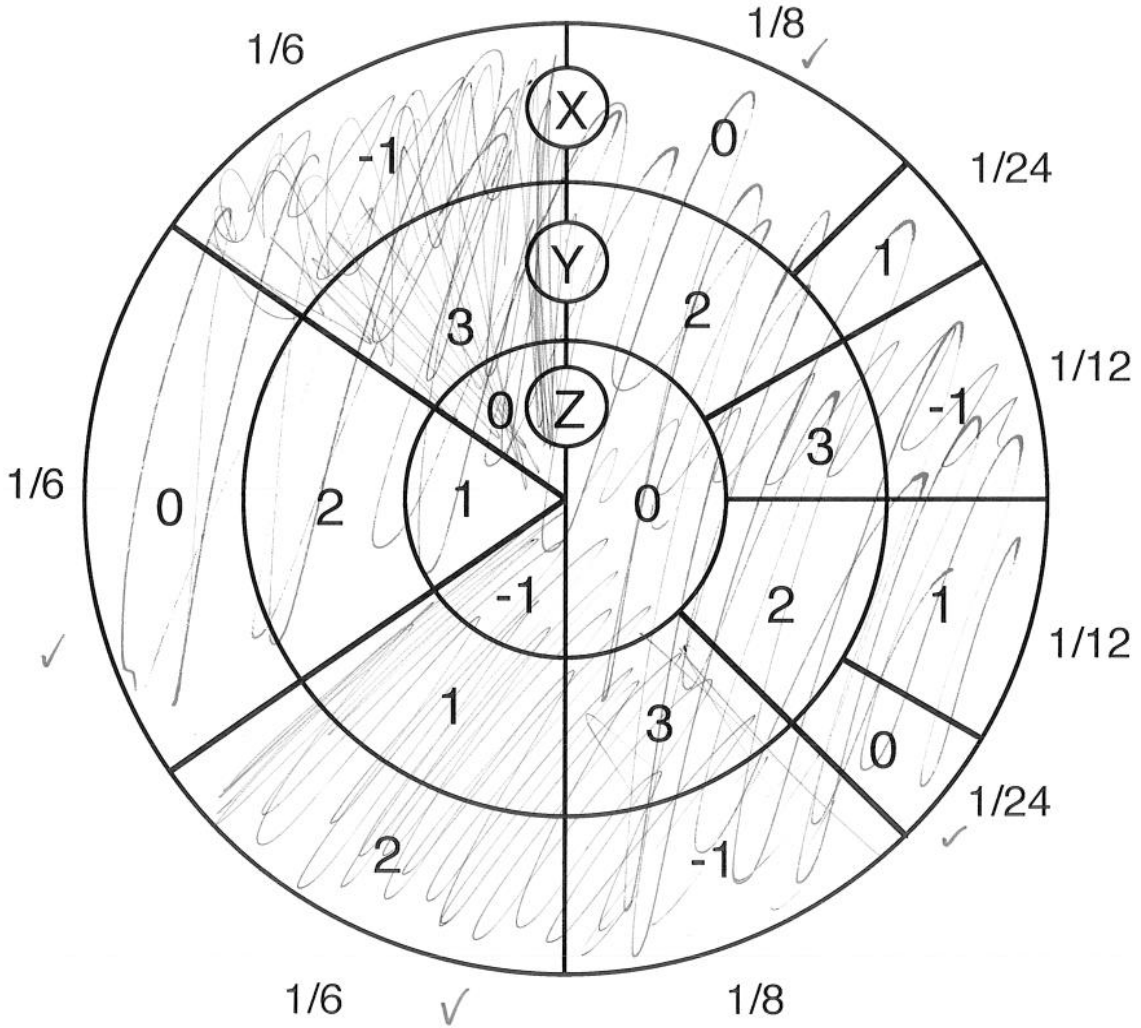
1. The wheel below represents the random variables X, Y and Z.



Calculate:

a) P(X=0) $= \frac{1}{8} + \frac{1}{24} + \frac{1}{6} = \frac{1}{3}$

b) P(X=1) $= \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$

✓c) P(Z=-1 or X=0) $= \frac{1}{6} + \frac{1}{6} + \frac{1}{24} + \frac{1}{8} = \frac{1}{2}$

d) P(Y=2) $= \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = \frac{11}{24}$

×e) P(Y=2 or X=0) $= \frac{11}{24}$

$P(Y=2) + P(X=0) - P(Y=2 \& X=0)$

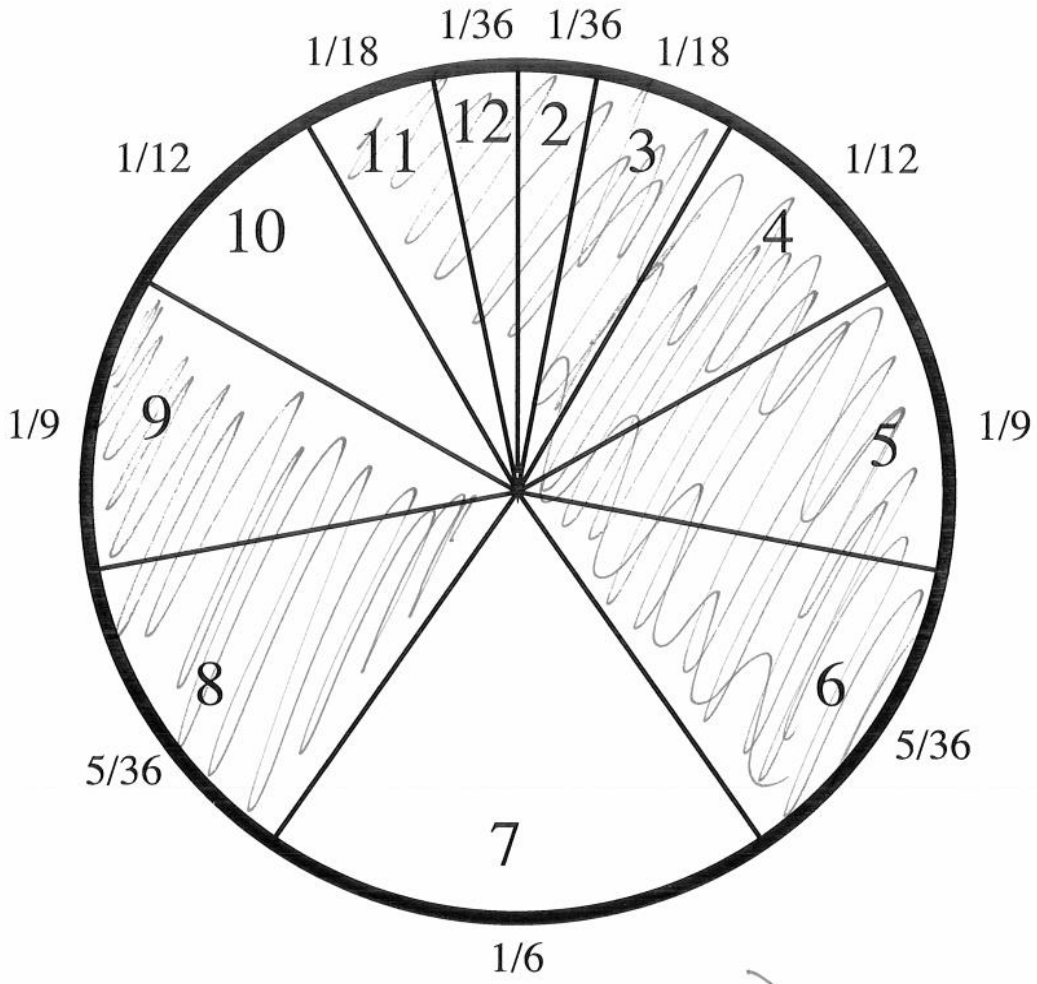
f) P(Y=2 and X=0) $= \frac{1}{3}$

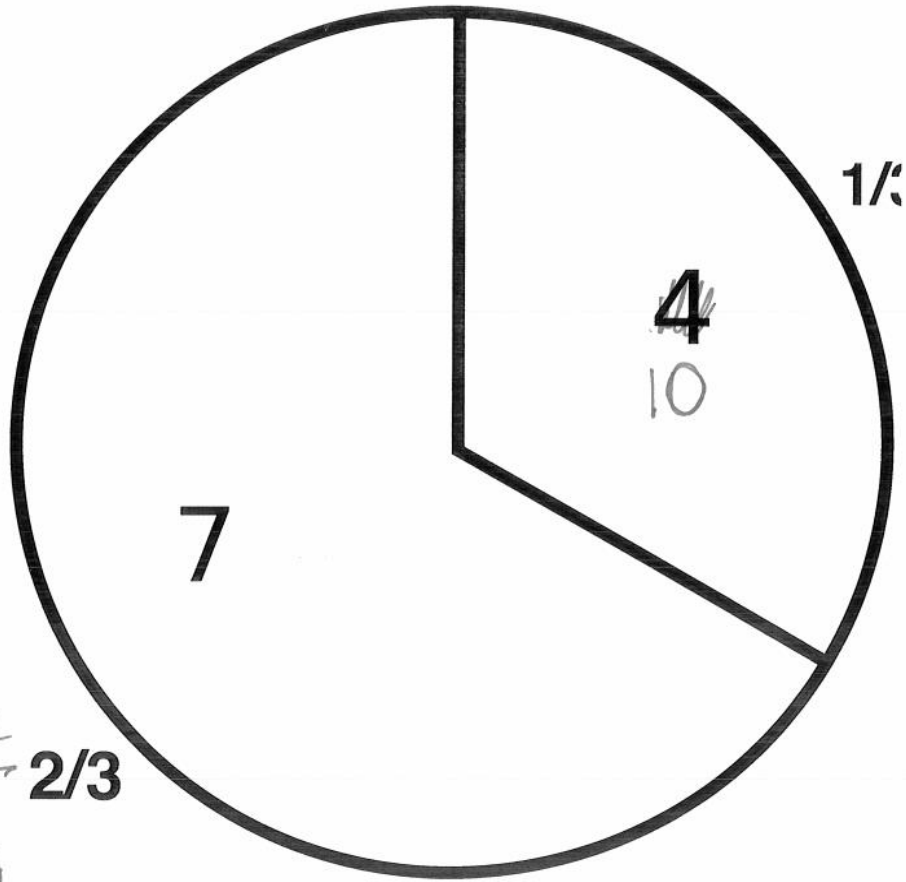
g) P(X=0 | Y=2) $= \frac{P(X=0 \& Y=2)}{P(Y=2)}$

$\frac{8}{11} = \frac{1/3}{11/24} = \frac{8}{11}$

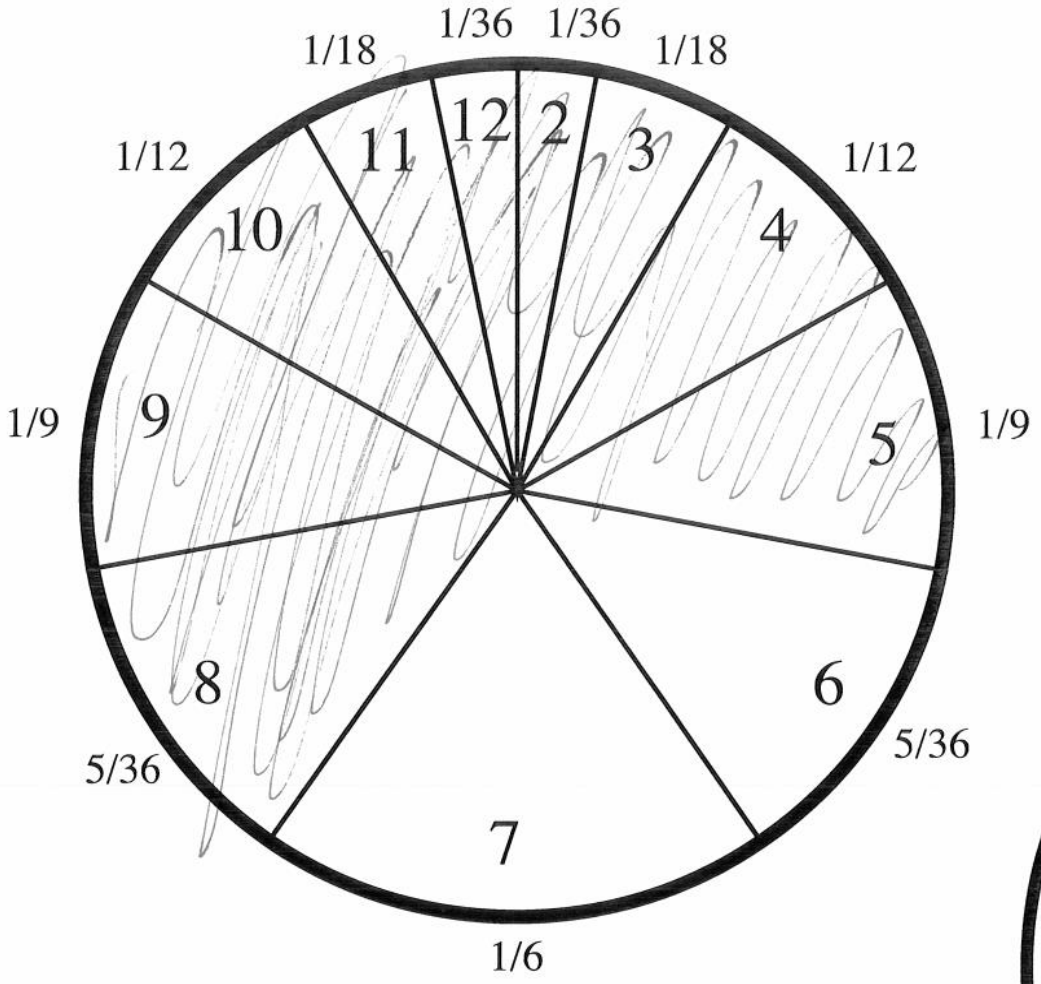
h) P(X=0 | Z=-1) $= 0$

i) P(X=2 | Z=-1) $= 1$

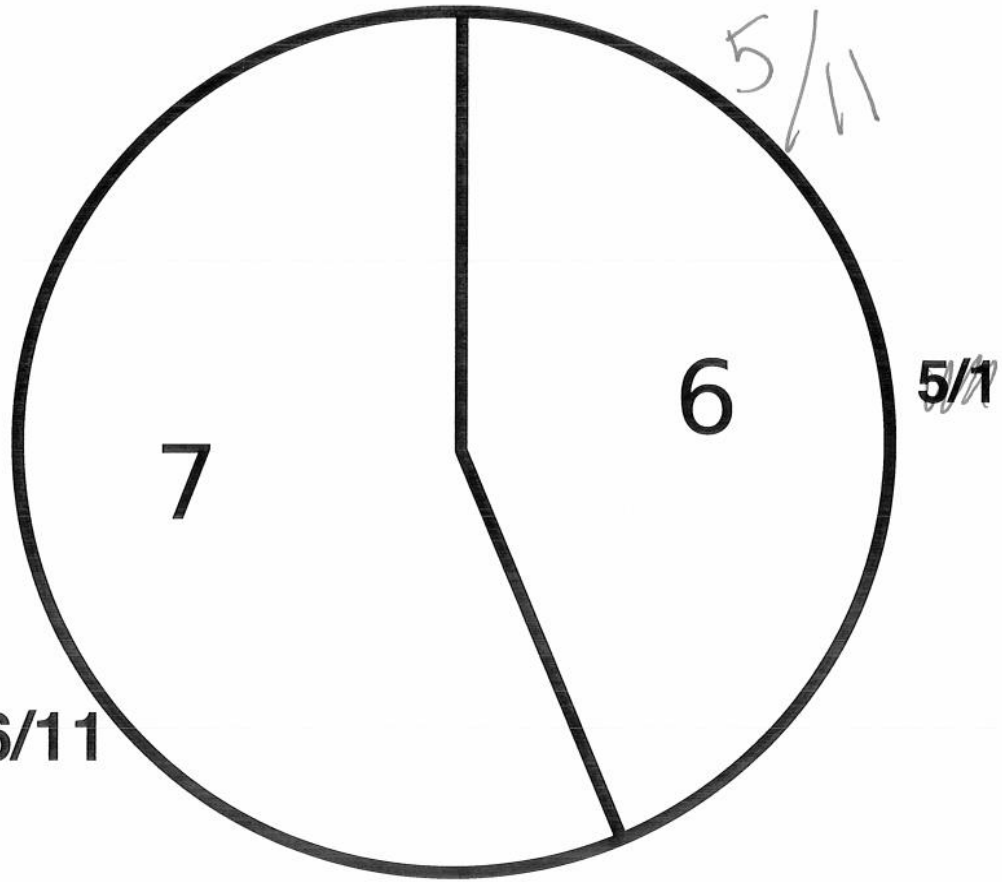


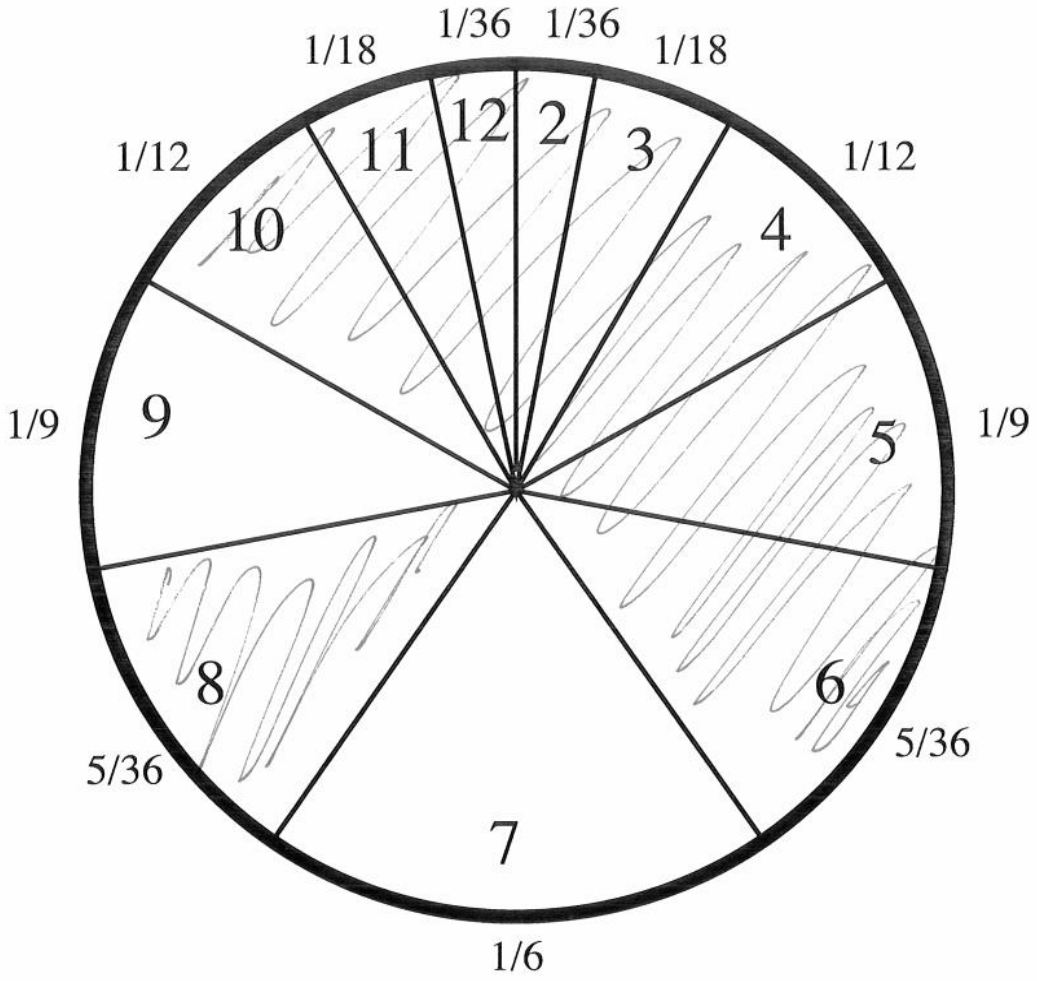


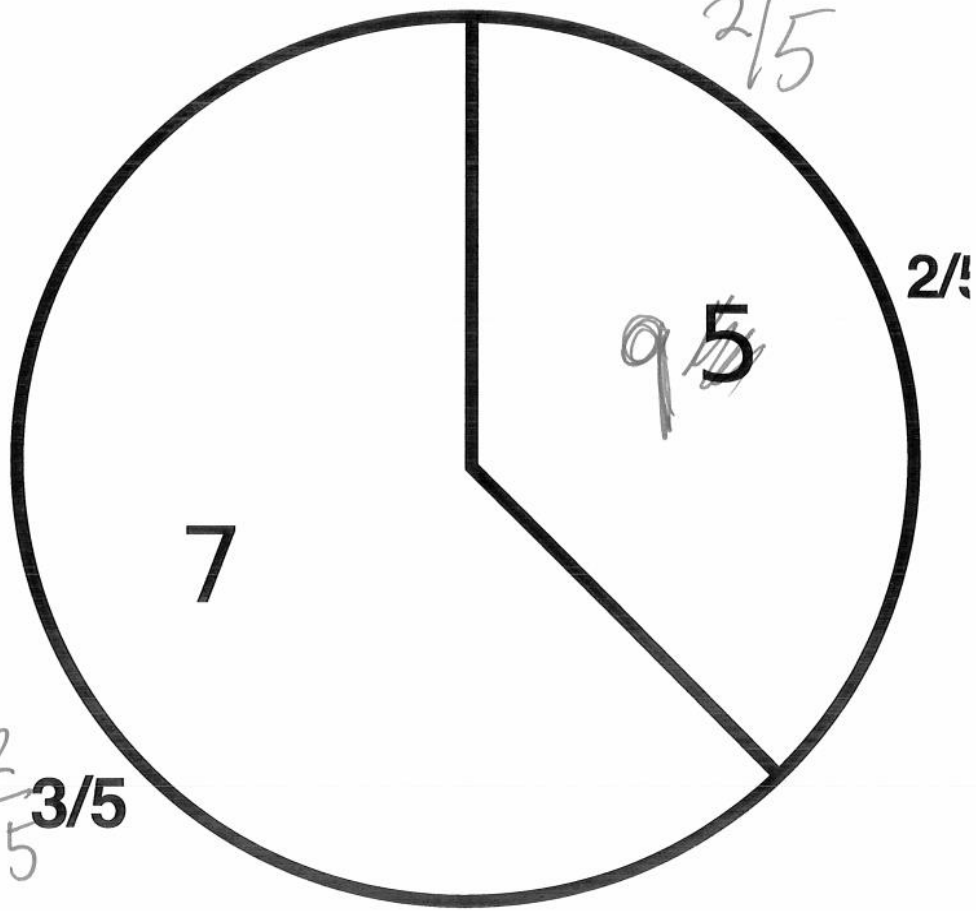
$$P(X=10 \mid X=7 \text{ OR } 10) = \frac{P(X=10 \ \& \ (X=7 \text{ OR } 10))}{P(X=7 \text{ OR } 10)} = \frac{1/12}{1/4} = 1/3$$



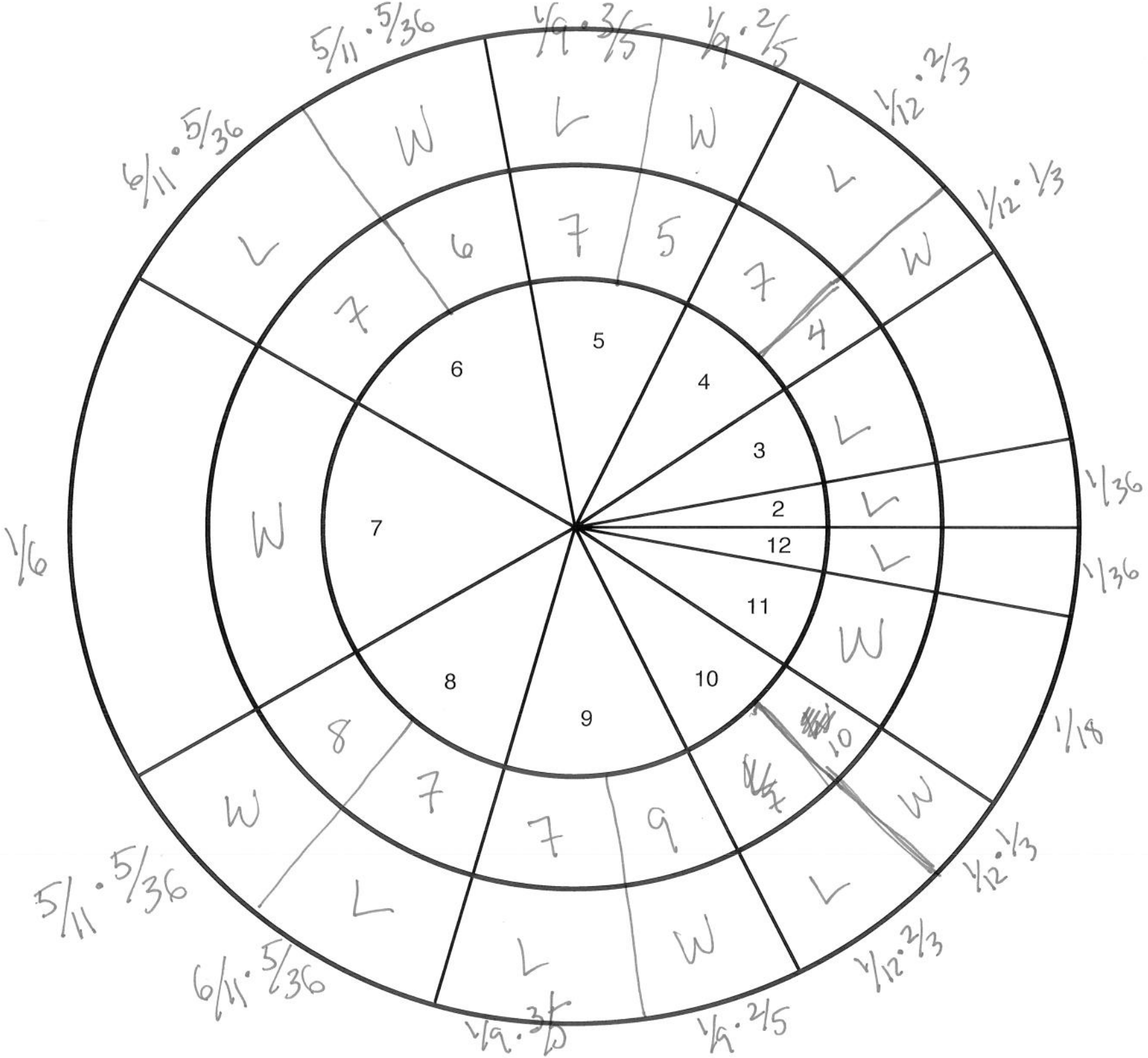
$$P(X=6 \mid X=6 \text{ OR } 7) = \frac{P(X=6 \;\&\; (X=6 \text{ OR } 7))}{P(X=6 \text{ OR } 7)} = \frac{5/36}{11/36}$$







$$P(X=9 \mid X=9 \text{ OR } 7) = \frac{P(X=9 \;\&\; (X=9 \text{ OR } 7))}{P(X=9 \text{ OR } 7)} = \frac{1/9}{5/18} = \frac{2}{5}$$


5/11 · 5/36
1/9 · 3/5
1/9 · 2/5
1/12 · 2/3
1/12 · 1/3
1/36
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1/18
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Example of conditional expectation of random variable

$$E(\text{a \$1 pass bet} \mid \text{the point is 6})$$

$$= +1 \cdot \frac{P(\text{win \& point is 6})}{P(\text{point is 6})} + (-1) \cdot \frac{P(\text{lose \& Point is 6})}{P(\text{point is 6})}$$

$$= +1 \cdot \frac{25/396}{5/36} + (-1) \cdot \frac{5/66}{5/36}$$

$$= +1 \cdot 5/11 + (-1) \cdot 6/11 = -1/11 = -.1111$$

E(a \$1 on the craps table)

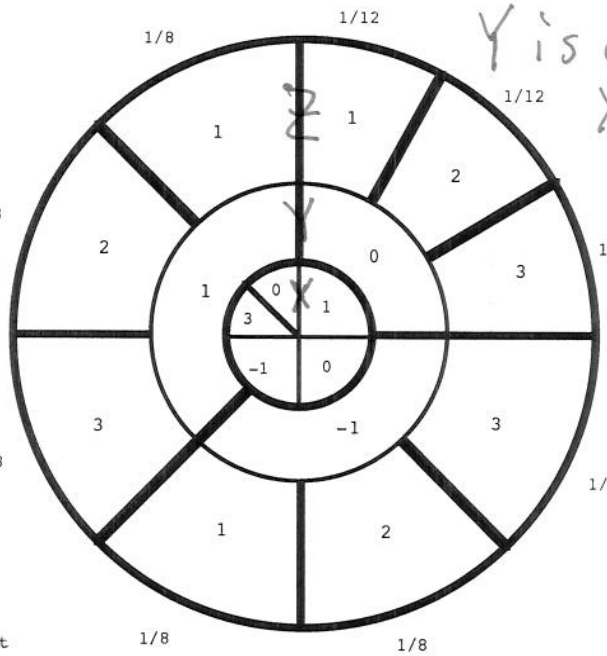
$+1 \cdot (.4929) + (-1) \cdot (.5071) = -.0142$

E(the field bet of \$1) =

$$+2 \cdot \left(\frac{1}{36} + \frac{1}{36}\right) + +1 \cdot \left(\frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36}\right) + (-1)\left(\frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36}\right)$$

$$= \frac{4}{36} + \frac{14}{36} - \frac{20}{36} = \frac{2}{36} - \frac{2}{36} = -.0556$$

1. Suppose that the random variables X,Y,Z are obtained by spinning the adjoining roulette, with X given by the innermost circle, Y given by the intermediate circle and Z given by the outer circle.



a) Calculate P[X=3,Y=1]

b) Are X and Y independent?
Are X and Z independent?
Are Y and Z independent?

c) Is any one of these variables dependent on the others?

d) Calculate E(X)

e) Calculate P[X=0|Z=1 or 3]

f) Calculate E(Z|X=0)

g) Suppose you bet 1 dollar that X+Z=2. Suppose you win w dollars if X+Z=2 and lose your dollar otherwise. What value should w have to make this a fair bet.
(Note that the bet is fair if on the average you can expect to break even)

X is dependent on Y & Z

Y is dependent on X & Z

Z is not dependent on X & Z

$w\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{12}\right)+(-1)\left(\frac{2}{3}\right)$

$= \frac{1}{3}w - \frac{2}{3} = 0 \quad w = 2$

2. As is now usual, we associate to letters of the alphabet the numbers, "space" is 26, $ is 27 and @ is 28. Decrypt the following message

JH@$S YDLEEROLBZTTPHXDTGGQAI

where the Hill matrix is

19 16

11 25

and we are working modulo 29.

3. The following message

DGDDF AFAAF XDFDF DAADA GVVVD
FAAXG GAVFA XAAVA DFFXG FDDDD

was encrypted using the ADFGVX system, with the attached ADFGVX square and the permutation

10 9 2 8 7 6 1 5 4 3

Unfortunately some of the letters in the square were lost as you can see.

Recover the original message.

	A	D	F	G	V	X
A	T	E	L	S	C	O
D	P	I	N			
F	D	F	G			
G	M	Q	U			
V	Y	Z	0			
X	4	5	6			