SOME CONNECTIONS BETWEEN ALGEBRAIC EXPRESSIONS AND SEQUENCES : PART I

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Let $A(q) = a_0 + a_1 q + a_2 q^2 + a_3 q^3 + \cdots$ and $B(q) = b_0 + b_1 q + b_2 q^2 + b_3 q^3 + \cdots$ where the integers a_i represent a sequence and b_i represent another sequence and q is a variable. That is, the coefficient of q^n in $A(q)$ is a_n and the coefficient of q^n in $B(q)$ is b_n. Recall that $1/(1-q) = 1+q+q^2+q^3+q^4+\cdots$ and by taking the derivative of this equation we can show that

$$1/(1-q)^2 = \frac{d}{dq}(1/(1-q)) = \frac{d}{dq}(1+q+q^2+q^3+q^4+\cdots) = 1+2q+3q^2+4q^3+\cdots.$$

What is the coefficient of q^n in the following expressions?

(1) $cA(q)$ (c is a constant here)
(2) $A(q) + B(q)$
(3) $q^m A(q)$
(4) $A(q) + q^{n-m} B(q)$
(5) $A'(q)$
(6) $A''(q)$
(7) $A(q)B(q)$
(8) $A(q)^2$
(9) $A(q)/(1-q^2)$
(10) $A(q)/(1-q)^2$
(11) $A'(q)/(1-q)$
(12) $(1+q)^k A(q)$
(13) $A(q)/(1-q)$
(14) $A(q^2)$
(15) $(1+q+q^2)A(q)$
(16) $\frac{1-q^k}{1-q} A(q)$
(17) $A(q) + A(-q)$
(18) $A(q) - A(-q)$