

**SOME CONNECTIONS BETWEEN SEQUENCES AND SETS OF OBJECTS:  
PART II**

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Say that  $a_0, a_1, a_2, \dots$  is a sequence of non-negative integers where  $a_n$  represents the number of “widgets of size  $n$ .” Assume similarly that  $b_n$  represents the number of “doodles of size  $n$ .” Below are a list of algebraic expressions labeled 1 through 13 and another list of combinatorial descriptions labeled (a) through (j). Match each one of the algebraic expressions with the combinatorial description. Here is the tough part: three of the equations do not have a combinatorial description. Write one for each of those.

- (1)  $a_n + b_n$
- (2)  $a_n + b_m$
- (3)  $\binom{n}{k} a_n$
- (4)  $\binom{n}{0} a_0 + \binom{n}{1} a_1 + \dots + \binom{n}{n-1} a_{n-1} + \binom{n}{n} a_n$
- (5)  $a_0 + a_1 + a_2 + \dots + a_n$
- (6)  $a_1 + 2a_2 + 3a_3 + \dots + na_n$
- (7)  $a_n b_n$
- (8)  $a_n^2$
- (9)  $a_1 a_2 \dots a_{n-1} a_n$
- (10)  $a_0 b_n + a_1 b_{n-1} + \dots + a_n b_0$
- (11)  $a_0 b_0 + a_1 b_1 + a_2 b_2 + \dots + a_n b_n$
- (12)  $\binom{n}{0} a_n b_0 + \binom{n}{1} a_{n-1} b_1 + \dots + \binom{n}{n-1} a_1 b_{n-1} + \binom{n}{n} a_0 b_n$
- (13)  $na_n$

- (a) The number of sequences of length  $n$  where the  $k^{\text{th}}$  element of the sequence is a widget of size  $k$ .
- (b) The number of pairs consisting of one widget and one doodle, both of the same size and each of size less than or equal to  $n$ .
- (c) The number of pairs whose first element is a widget of size  $n$  and whose second element is a doodle of the same size.
- (d) The number of pairs consisting of a subset of the numbers 1 through  $n$  and a widget which is the same size as the subset.
- (e) The number of pairs which is either a widget of size  $n$  and the number 1 or it is a doodle of size  $n$  and the number 2.
- (f) The number of pairs whose first element is a subset of the integers 1 through  $n$  of size  $k$  and whose second element is a widget of size  $n$ .
- (g) The number of pairs consisting of two widgets both of the same size and both of size  $n$ .
- (h) The number of pairs consisting of a widget and a doodle such that the size of the widget plus the size of the doodle is  $n$ .
- (i) The number of triples consisting of a widget and a doodle and a subset of the integers 1 through  $n$  such that the widget and the subset are of the same size and the size of the widget plus the size of the doodle must be  $n$ .
- (j) The number of pairs consisting of a widget whose size is less than or equal to  $n$  and an integer between 1 and the size of the widget.