LIST OF QUESTIONS ASSIGNED DURING FALL 2003 FOR MATH 5020

There was a whole chapter of counting questions that I photocopied from a book. Each week I asked you to do roughly 25 of those problems until we finished them. I obviously didn't expect you to do 25 each week but I do expect that your journals will have a random sample of these questions in them, and not just the easy ones (you will need to write the full statement of the questions in your journal). I just expect you to do enough of these questions until you become an expert at this type of problem.

There were a couple of "Prove by counting a set in two different ways:" first question:

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

one step harder:

$$\binom{n+2}{k+2} = \binom{n}{k} + 2\binom{n}{k+1} + \binom{n}{k+2}$$

The general case:

$$\binom{n+r}{k+r} = \binom{n}{k}\binom{r}{0} + \binom{n}{k+1}\binom{r}{1} + \cdots + \binom{n}{k+r}\binom{r}{r}$$

first question:

$$\binom{n+k+1}{k} = \binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+k}{k}$$

one step harder:

$$\binom{n+k+2}{k} = \binom{n}{0}(k+1) + \binom{n+1}{1}k + \binom{n+2}{2}(k-1) + \cdots + \binom{n+k}{k}$$

The general case:

$$\binom{n+k+m}{k} = \binom{n}{0}\binom{k+m-1}{m-1} + \binom{n+1}{1}\binom{k+m-2}{m-1} + \binom{n+2}{2}\binom{k+m-3}{m-1} + \cdots + \binom{n+k}{k}\binom{m-1}{m-1}$$

I don't think that i asked you to prove the general case of the second problem but I did write it down in class.

The following problems I assigned to be done on the FORUM. I may or may not have mentioned them in class:
Section 1.1 # 3,4,5,6,8,9,11,12,13,14,15,16,18
Section 1.2 #2,4,5
Section 2.2 # 2,3,6,9,12
All of the rest of the FORUM questions were taken from the photocopied questions from the book "Applied Combinatorics" by Alan Tucker.

In addition, I have mentioned in my notes that I have assigned the following problems from the text "Number Theory" by George Andrews (numbers in parentheses were probably not ever assigned,

but you can do them anyway if you want):
Section 1.1 # 1-18
Section 1.2 # (1), 2, (3), 4, 5
Section 2.1 # (6), 7
Section 2.2 # 2,3, 6, (8), 9, 12
Next I think that I am missing a bunch of questions that I asked you to look at since the next time that I have anything listed in my notes is:
Section 5.2 # 3, 16, 19

From January 5, 2004:
How many rearrangements are there of the letters of the word 'GREATGRANDFATHER' where the word 'GREAT' appears consecutively (in that order).

From January 12, 2004:
I gave an RSA problem that was to be done on the computer. This is posted on the web page.

In Lotto 6/49, what is the probability of holding a ticket with 4 of the 6 winning numbers correct and the bonus?

From January 19, 2004:
Buy a lottery ticket.

Verify that $d(n)$ and $\sigma(n)$ are multiplicative functions by first proving that $d(p_1^{\alpha_1} \cdots p_k^{\alpha_k}) = (\alpha_1 + 1)(\alpha_2 + 1) \cdots (\alpha_k + 1)$ (note: this one you should be able to prove by the multiplication principle) and that $\sigma(p_1^{\alpha_1} \cdots p_k^{\alpha_k}) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1}-1}{p_i-1}$ (note: prove this by induction on k and the proof is in the book). Convince yourself that these formulas show that d and σ are multiplicative.

Go to your favorite search engine and enter "Encyclopedia of Integer Sequences" and enter into the database "1,1,3,4,11,..." which is what we calculated to be the first five number of if the sequence $a_n :=$ then number of words of 1s and 2s with at least as many 1s as 2s. Use the database to find the entry that we are looking for (you might need to calculate an extra term or two) and get the formula for this sequence.

Let $a_i =$ the number of widgets of size i and $b_i =$ the number of doodles of size i (we had been talking about the concept of sequences representing the number of objects in some set and "widgets" and "doodles" are my words to represent an arbitrary set of objects).
Explain what $a_i + b_i$ represents.
Explain what $a_i b_j$ represents.
Explain what $a_n b_0 + a_{n-1} b_1 + a_{n-2} b_2 + \cdots + a_1 b_{n-1} + a_0 b_n$ represents.
Note: what I am looking for is that you provide me with a description of something so that if someone applies combinatorial techniques to your description to count the number of objects they will get the expressions above.

From January 26, 2004:
Problem #15 in section 1.1. Please look on the web page for my "hint" on this one.

If anyone can help me out with the questions that I am missing either e-mail me at `zabrocki@mathstat.yorku.ca` or post the corrections on the FORUM.