UNEXAM #2

ASSIGNED: JANUARY 28, 2009 DUE: FEBRUARY 11, 2009

Remember that the important aspect of this assignment is not the answer, but the solution. I am looking for a simple, clear, thoughtful, direct, short explanation.

The assignment is due February 11.

(1) What is the generating function for the number of integral solutions of the equation

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = n$$

where each of the x_i are non-negative integers?
Note: this is a sequence of numbers dependent on a parameter n. Answer:

$$\frac{1}{(1-q)(1-q^2)(1-q^3)(1-q^4)(1-q^5)}$$

(2) Using generating the generating functions for the Fibonnaci numbers are $F(q) = \frac{1}{1-q-q^2} = \sum_{n\geq 0} F_{n+1}q^n$ explain why for $n \geq 0$:

$$F_{n+7} - F_{n+1} = 4F_{n+4}$$

Answer: Because the generating function for $q^6 \sum_{n\geq 0} F_{n+7}q^n = \frac{1}{1-q-q^2} - 1 - q - 2q^2 - 3q^3 - 5q^4 - 8q^5 = q^6 \frac{(13+8\,q)}{1-q-q^2}$ and the generating function for $q^3 \sum_{n\geq 0} F_{n+4}q^n = \frac{1}{1-q-q^2} - 1 - q - 2q^2 = q^3 \frac{(3+2\,q)}{1-q-q^2}$.