Real world applications of representation theory of non-abelian groups

(Subtitle: Representation theorists WILL rule the world one day just you wait :-)

This page, written at the suggestion of the Director of Research at the USNA, Prof. Reza Malek-Madani, describes some applications of representation theory of non-abelian groups to various fields and gives some references. However, I am entirely to blame for the subtitle.

- *Engineering*.
  - *Tensegrity - the design of "strut-and-cable" constructions*.
    Want to build a building with cables and struts but don't know representation theory? Check out these references:
    - R. Connelly and A. Back, "Mathematics and tensegrity", Amer Scientist, April-May 1998, pages 142-151
    - symmetric tensegrities
  - *Telephone network designs*.
    This is the information age with more and more telephone lines needed every day. Want to reach out and touch someone? You need representation theory.
    - F. Bien, "Construction of telephone networks by group representations", Notices A. M. S. **36**(1989)5-22
  - *Nonlinear network problems*.
    This is cheating a little since the works in the reference below really use the theory of Lie groups instead of representation theory itself. Still, there is a tangential relation at least between representation theory of Lie groups and the solution to certain nonlinear network problems.
    - C. Desoer, R. Brockett, J. Wood, R. Hirshorn, A. Willsky, G. Blankenship, **Applications of Lie group theory to nonlinear network problems**, (Supplement to IEEE Symposium on Circuit Theory, 1974), Western Periodicals Co., N. Hollywood, CA, 1974
  - *Control theory*.
    - R. W. Brockett, "Lie theory and control systems defined on spheres", SIAM J on Applied Math **25**(1973) 213-225
  - *Robotics*.
    The future is not in plastics (see the movie "The Graduate") but in robotics. How do you figure out their movements before building them? You guessed it, using representation theory.
    - G. Chirikjian, "Determination and synthesis of discretely actuated manipulator workspaces using harmonic analysis", in Advances in Robotic Kinematics, **5**, 1996, Springer-Verlag
    - G. Chirikjian and I. Ebert-Uphoff, "Discretely actuated manipulator workspace generation by closed-form convolution", in ASME Design Engineering Technical Conference, August 18-22 1996
  - *Radar design*.
    W. Schempp, **Harmonic analysis on the Heisenberg nilpotent Lie group, with applications to**

signal theory, Longman Scientific & Technical, New York (Copublished in the U.S. with Wiley), 1986.

- *Antenna design*.
  B. Hassibi, B. Hochwald, A. Shokrollahi, W. Sweldens, ``Representation theory for high-rate multiple antenna code design," 2000 preprint (see A. Shokrollahi's site for similar works).
- *Design of stereo systems*.
  We're talkin' quadrophonic state-of-the-art.
  - K. Hannabus, "Sound and symmetry", Math. Intelligencer, **19**, Fall 1997, pages 16-20
- *Coding theory*. Interesting progress in coding theory has been made using group theory and representation theory.
  - F. MacWilliams and N. Sloane, **The Theory of Error-Correcting Codes**, North-Holland/Elsevier, 1993 (8th printing)
  - I. Blake and R. Mullin, **Mathematical Theory of Coding**, Academic Press, 1975
  - F. Harald, "Enumeration of isometry-classes of linear (n,k)-codes over GF(q) in SYMMETRICA", Bayreuther Math. Scriften **49**(1995)215-223
  - J.-P. Tillich and G. Zemor, "Optimal cycle codes constructed from Ramanujan graphs," SIAM J on Disc. Math. **10**(1997)447-459
  - H. Ward and J. Wood, "Characters and the equivalence of codes," J. Combin. Theory A **73**348-352
  - J. Lafferty and D. Rockmore, "Spectral Techniques for Expander Codes" , (Extended Abstract) 1997 Symposium on Theory of Computation (available at Dan Rockmore's web page)

- *Mathematical physics*. Any complete list of books and papers in this field which use representation theory would be much too long for the limited goal we have here (which is simply to list some real-world applications). A small selection is given below.
  - *Differential equations (such as the heat equation, Schrodinger wave equation, etc)*.
    M. Craddock, "The symmetry groups of linear partial differential equations and representation theory, I" J. Diff. Equations **116**(1995)202-247
  - *Mechanics*.
    - D.H. Sattinger, O.L. Weaver, **Lie Groups and Algebras With Applications to Physics, Geometry, and Mechanics (Applied Mathematical Sciences, Vol 61)** , Springer Verlag, 1986
    - Johan Belinfante, "Lie algebras and inhomogeneous simple materials", SIAM J on Applied Math **25**(1973)260-268
  - *Models for elementary particles*.
    - Howard Georgi, **Lie Algebras in Particle Physics**, Addison-Wesley, 1995
    - J. Baez, " This week's finds in mathematical physics - week 119," posted to sci.math.research on 4-13-1998
  - *Quantum mechanics*.
    - Eugene Wigner, "Reduction of direct products and restriction of representations to subgroups: the everyday tasks of the quantum theorists", SIAM J on Applied Math **25**(1973) 169-185
    - V. Vladimirov, I. Volovich, and E. Zelenov, "Spectral theory in p-adic quantum mechanics and representation theory," Soviet Math. Doklady **41**(1990)40-44
  - *p-adic string theory*.
    - Y. Manin, "Reflections on arithmetical physics," in **Conformal invariance and string theory** Academic Press, 1989, pages 293-303
    - V. Vladimirov, I. Volovich, and E. Zelenov, **p-adic analysis and mathematical physics**, World Scientific, 1994
    - V. Vladimirov, "On the Freund-Witten adelic formula for Veneziano amplitudes," Letters in Math. Physics **27**(1993)123-131
- *Mathematial chemistry*.
  - *Spectroscopy*.
    B. Judd, "Lie groups in Atomic and molecular spectroscopy", SIAM J on Applied Math **25**(1973) 186-192
  - *Crystallography*.

        - G. Ramachandran and R. Srinivasan, **Fourier methods in crystallography**, New York, Wiley-Interscience, 1970.
        - T. Janssen, **Crystallographic groups**, North-Holland Pub., London, 1973.
        - J. Zak, A. Casher, M. Gluck, Y. Gur, **The irreducible representations of space groups**, W. A. Benjamin, Inc., New York, 1969.
        - The Topology of Crystallographic Groups and Simple Crystal Structures
        - Crystallography 101 (framed html web pages, click on **About Symmetry and Space Groups**)
    - *Molecular strucure of the Buckyball*.
        - F. Chung and S. Sternberg, "Mathematics and the buckyball", American Scientist **83**(1993)56-71
        - F. Chung, B. Kostant, and S. Sternberg, "Groups and the buckyball", in **Lie theory and geometry**, (ed. J.-L. Brylinski et al), Birkhauser, 1994
        - G. James, "The representation theory for the Buckminsterfullerene," J. Alg. **167**(1994)803-820
- *Knot theory* (which, in turn, has applications to modeling DNA) uses representation theory.
  F. Constantinescu and F. Toppan, "On the linearized Artin braid representation," J. Knot Theory and its Ramifications, **2**(1993)
- The *Riemann hypothesis*. Think you're going to solve the Riemann hypothesis without using representation theory? Get real!
  A. Connes, "Formule de traces en geometrie non-commutative et hypothese de Riemann", C. R. Acad. Sci. Paris **323** (1996)1231-1236

  (Some may argue that this is not a real-world application but we refer to Barry Cipra's article, "Prime Formula Weds Number Theory and Quantum Physics," Science, 1996 December 20, **274**, no. 5295, page 2014, in Research News.)

- *Circuit design, statistics, signal processing, ...* See the survey paper D. Rockmore, "Some applications of generalized FFTs" in Proceedings of the DIMACS Workshop on Groups and Computation, June 7-10, 1995 eds. L. Finkelstein and W. Kantor, (1997) 329--369. (available at Dan Rockmore's web page)
- *Vision* See the survey paper by Jacek Turski: Geometric Fourier Analysis of the Conformal Camera for Active Vision SIAM Review, Volume 46 Issue 2 pages 230-255, © 2004 Society for Industrial and Applied Mathematics. and J. Turski, Geometric Fourier Analysis for Computational Vision, JFAA 11, 1-23, 2005.

Please send additions or comments to David Joyner .

Created 6-13-98 and last updated 3-31-2006