Course Description

Math 6161- Algebraic Combinatorics : Symmetric Functions

Course Description:

Gambelli proved in 1902 that any algebra that has a basis that satisfies a Pieri like multiplication rule is isomorphic to a quotient of the symmetric functions. For this reason questions in many seemingly unrelated fields can be reduced to statements in this algebra and resolved using computational techniques that we use to study the symmetric functions. The algebraba of symmetric functions is fundamental because it provides a common language in which to understand questions in geometry, combinatorics, algebra, and representation theory.

This course begins with an introduction to the representation theory of the symmetric group to provide some motivation and introduce notation. The symmetric functions are defined as a Hopf algebra and the structure of this algebra is studied from a combinatorial perspective by examining the fundamental bases and the change of basis coefficients. Only later is this algebra shown to be isomorphic to the ring of polynomials which are invariant under permutations of the variables. The use of Maple to do computation will be encouraged by the introduction of the package SF by John Stembridge.

This course meets twice a week, Tuesday/Thursday from 2:30-5:30 in S101A Ross. Part of this time will be lecture/problem session and we will spend part of this time in a computer lab (where depends on the number of students attending the class).

The representation theory component of the course will follow "The Symmetric Group : Representations, Combinatorial Algorithms & Symmetric Functions" by Bruce Sagan. The part of the course that covers symmetric functions will follow notes that I am in the process of writing that approach the subject of symmetric functions from the *plethystic notation* perspective.

Prof. Mike Zabrocki
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Course Evaluation:

Midterm: 30%
Homework and Labs: 30%
Final Exam: 40%

Check the schedule for dates of the midterm/homeworks/final.

Schedule

Math 6161- Algebraic Combinatorics : Symmetric Functions

Course Description
Handouts
Computer Labs
Links
Schedule
References

May/June						
S	Su	M	Tu	W	Th	F
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31	1	2	3	4	5	6
7	8	9	10	11	12	13

class day = ☐ midterm = ☐ final = ☐

I will try to keep a diary/schedule of the topics we cover in the class and roughly the sections that they correspond to in the text. S = Segan the Symmetric Group, N = Class notes.
The "Labs" that are referred to below are just Maple worksheets which will have programs and examples which we discuss in class. These can be found on the 'Computer Lab' page.

May 6	The Symmetric group, permutations, cycle structure, matrix representations, G-modules, reducibility S1.1-1.4 Lab 1 Exercises (first edition!): 1.2, 1.5.a (see below)
May 8	Complete reducibility and Maschke's theorem, Schur's Lemma
May 13	

Exercises:

- (1.2) If a group G acts on a set **S** and s is in **S**, then the *stablizer* of s is $\mathbf{G}_s$ = { g in G such that g s = s }. The *orbit* if s is the set $\mathbf{O}_s$ = { g s over all possible g in G}.
  (a) Prove that $\mathbf{G}_s$ is a subgroup of G.
  (b) Find a bijection between cosets of $G/\mathbf{G}_s$ and elements of $\mathbf{O}_s$.
  (c) Show that $|\mathbf{O}_s| = |G|/|\mathbf{G}_s|$ and use this to show that the number of elements conjugate to an element g is equal to $|G|/|Z_s|$ where Z_s = { h in G such that h g h^(-1) = g }
- (1.5) If X is a matrix representation of a group G, then its kernel is the set N = { g in G | X(g) = I }. A representation is faithful if it is one-to-one.
  (a) Show that N is a normal subgroup of G and find a condition of on N equivalent to the representation being faithful.