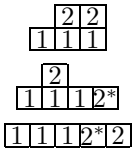
1. Conjectured tableaux rank functions for $n = 5$

1.1. $G_\lambda[X;q]$ for λ strict partition of 5.

$$G^q_{3,2} = 2\,q^2 Q_5 + 2\,q Q_{4,1} + Q_{3,2}$$
$$G^q_{4,1} = 2\,q Q_5 + Q_{4,1}$$
$$G^q_5 = Q_5$$

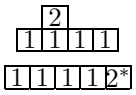
1.2. Conjectured tableaux rank function of content $(3,2)$.

$$G^q_{3,2} = 2\,q^2 Q_5 + 2\,q Q_{4,1} + Q_{3,2}$$



1.3. Conjectured tableaux rank function of content $(4,1)$.

$$G^q_{4,1} = 2\,q Q_5 + Q_{4,1}$$



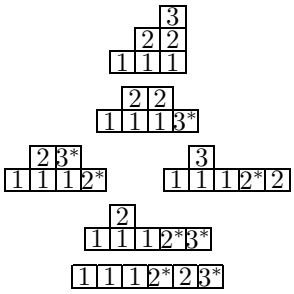
2. Conjectured tableaux rank functions for $n = 6$

2.1. $G_\lambda[X;q]$ for λ strict partition of 6.

$$G^q_{3,2,1} = 4\,q^4 Q_6 + \left(4\,q^3 + 2\,q^2\right) Q_{5,1} + \left(4\,q^2 + 2\,q\right) Q_{4,2} + Q_{3,2,1}$$
$$G^q_{4,2} = 2\,q^2 Q_6 + 2\,q Q_{5,1} + Q_{4,2}$$
$$G^q_{5,1} = 2\,q Q_6 + Q_{5,1}$$
$$G^q_6 = Q_6$$

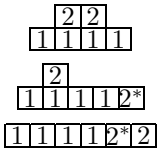
2.2. Conjectured tableaux rank function of content $(3,2,1)$.

$$4\,q^4 Q_6 + \left(2\,q^2 + 4\,q^3\right) Q_{5,1} + \left(2\,q + 4\,q^2\right) Q_{4,2} + Q_{3,2,1}$$



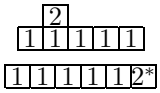
2.3. Conjectured tableaux rank function of content $(4,2)$.

$$G^q_{4,2} = 2\,q^2 Q_6 + 2\,q Q_{5,1} + Q_{4,2}$$



2.4. Conjectured tableaux rank function of content $(5,1)$.

$$G^q_{5,1} = 2\,q Q_6 + Q_{5,1}$$



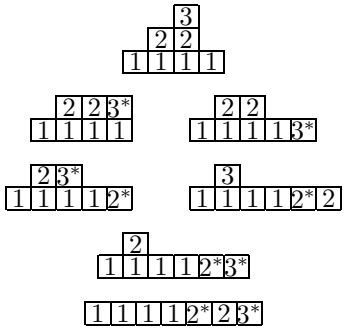
3. Conjectured tableaux rank functions for $n = 7$

3.1. $G_\lambda[X;q]$ for λ strict partition of 7.

$$G^q_{4,2,1} = 4\,q^4 Q_7 + \left(4\,q^3 + 2\,q^2\right) Q_{6,1} + \left(4\,q^2 + 2\,q\right) Q_{5,2} + 2\,qQ_{4,3} + Q_{4,2,1}$$
$$G^q_{4,3} = 2\,q^3 Q_7 + 2\,q^2 Q_{6,1} + 2\,qQ_{5,2} + Q_{4,3}$$
$$G^q_{5,2} = 2\,q^2 Q_7 + 2\,qQ_{6,1} + Q_{5,2}$$
$$G^q_{6,1} = 2\,qQ_7 + Q_{6,1}$$
$$G^q_7 = Q_7$$

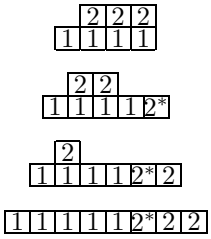
3.2. Conjectured tableaux rank function of content $(4,2,1)$.

$$G^q_{4,2,1} = 4\,q^4 Q_7 + \left(4\,q^3 + 2\,q^2\right) Q_{6,1} + \left(2\,q + 4\,q^2\right) Q_{5,2} + 2\,qQ_{4,3} + Q_{4,2,1}$$



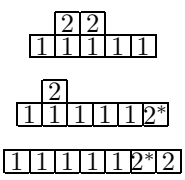
3.3. Conjectured tableaux rank function of content $(4,3)$.

$$G^q_{4,3} = 2\,q^3 Q_7 + 2\,q^2 Q_{6,1} + 2\,qQ_{5,2} + Q_{4,3}$$



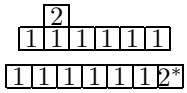
3.4. Conjectured tableaux rank function of content $(5,2)$.

$$G^q_{5,2} = 2\,q^2 Q_7 + 2\,qQ_{6,1} + Q_{5,2}$$



3.5. **Conjectured tableaux rank function of content** $(6,1)$**.**

$$G^q_{6,1} = 2\,qQ_7 + Q_{6,1}$$



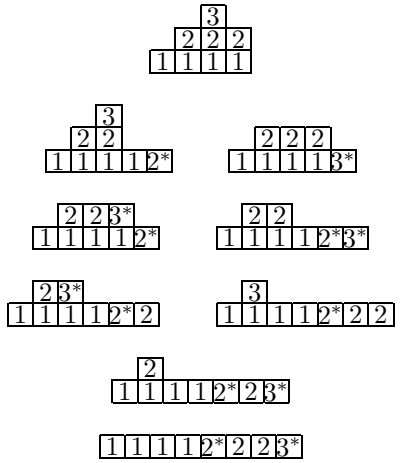
4. Conjectured tableaux rank functions for $n = 8$

4.1. $G_\lambda[X;q]$ for λ strict partition of 8.

$$G^q_{4,3,1} = 4\,q^5 Q_8 + \left(2\,q^3 + 4\,q^4\right) Q_{7,1} + \left(4\,q^3 + 4\,q^2\right) Q_{6,2} + \left(4\,q^2 + 2\,q\right) Q_{5,3} + 2\,q Q_{5,2,1} + Q_{4,3,1}$$
$$G^q_{5,2,1} = 4\,q^4 Q_8 + \left(2\,q^2 + 4\,q^3\right) Q_{7,1} + \left(4\,q^2 + 2\,q\right) Q_{6,2} + 2\,q Q_{5,3} + Q_{5,2,1}$$
$$G^q_{5,3} = 2\,q^3 Q_8 + 2\,q^2 Q_{7,1} + 2\,q Q_{6,2} + Q_{5,3}$$
$$G^q_{6,2} = 2\,q^2 Q_8 + 2\,q Q_{7,1} + Q_{6,2}$$
$$G^q_{7,1} = 2\,q Q_8 + Q_{7,1}$$
$$G^q_8 = Q_8$$

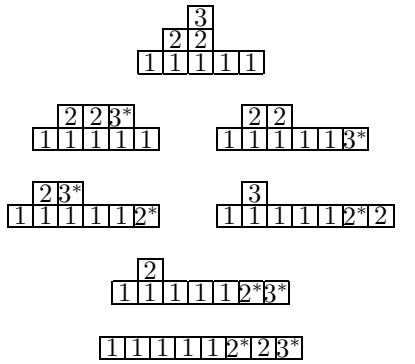
4.2. Conjectured tableaux rank function of content $(4,3,1)$.

$$G^q_{4,3,1} = 4\,q^5 Q_8 + \left(2\,q^3 + 4\,q^4\right) Q_{7,1} + \left(4\,q^2 + 4\,q^3\right) Q_{6,2} + \left(2\,q + 4\,q^2\right) Q_{5,3} + 2\,q Q_{5,2,1} + Q_{4,3,1}$$



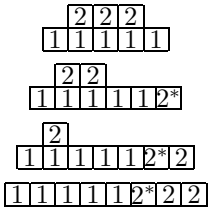
4.3. Conjectured tableaux rank function of content $(5,2,1)$.

$$G^q_{5,2,1} = 4\,q^4 Q_8 + \left(2\,q^2 + 4\,q^3\right) Q_{7,1} + \left(4\,q^2 + 2\,q\right) Q_{6,2} + 2\,q Q_{5,3} + Q_{5,2,1}$$



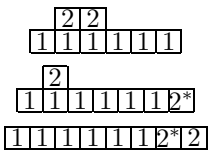
4.4. **Conjectured tableaux rank function of content** $(5,3)$**.**

$$G^q_{5,3} = 2\,q^3 Q_8 + 2\,q^2 Q_{7,1} + 2\,q Q_{6,2} + Q_{5,3}$$



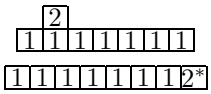
4.5. **Conjectured tableaux rank function of content** $(6,2)$**.**

$$G^q_{6,2} = 2\,q^2 Q_8 + 2\,q Q_{7,1} + Q_{6,2}$$



4.6. **Conjectured tableaux rank function of content** $(7,1)$**.**

$$G^q_{7,1} = 2\,q Q_8 + Q_{7,1}$$



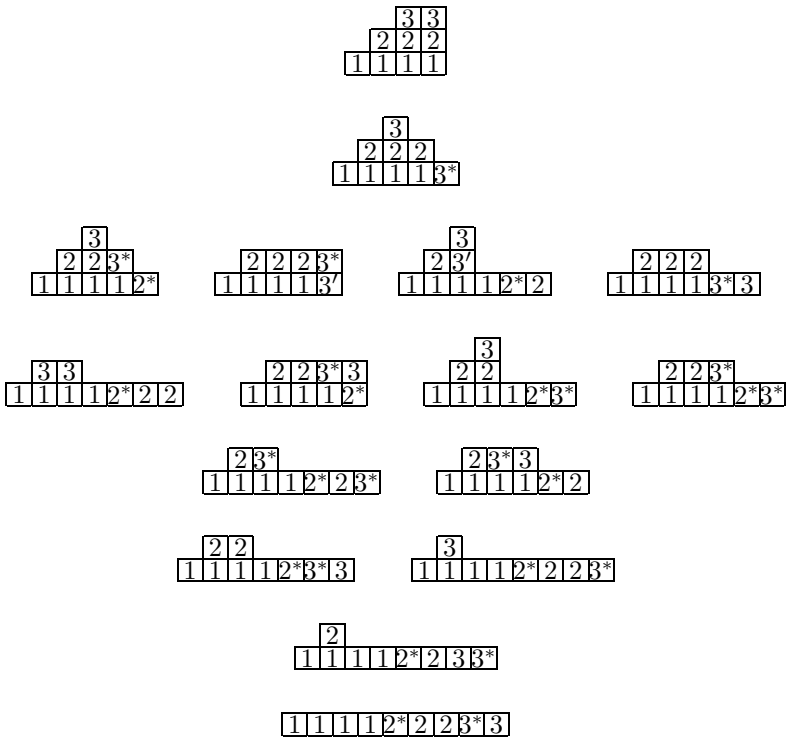
5. Conjectured rank functions for $n = 9$

5.1. $G_\lambda[X;q]$ for λ strict partition of 9.

$$
\begin{aligned}
G^q_{4,3,2} &= 4\,q^7 Q_9 + \left(4\,q^5 + 4\,q^6\right) Q_{8,1} + \left(2\,q^3 + 8\,q^4 + 4\,q^5\right) Q_{7,2} + \left(8\,q^3 + 4\,q^4 + 2\,q^2\right) Q_{6,3} \\
&\quad + \left(4\,q^3 + 2\,q^2\right) Q_{6,2,1} + \left(4\,q^3 + 2\,q^2\right) Q_{5,4} + \left(4\,q^2 + 2\,q\right) Q_{5,3,1} + Q_{4,3,2} \\
G^q_{5,3,1} &= 4\,q^5 Q_9 + \left(4\,q^4 + 2\,q^3\right) Q_{8,1} + \left(4\,q^3 + 4\,q^2\right) Q_{7,2} + \left(4\,q^2 + 2\,q\right) Q_{6,3} \\
&\quad + 2\,q Q_{6,2,1} + 2\,q Q_{5,4} + Q_{5,3,1} \\
G^q_{5,4} &= 2\,q^4 Q_9 + 2\,q^3 Q_{8,1} + 2\,q^2 Q_{7,2} + 2\,q Q_{6,3} + Q_{5,4} \\
G^q_{6,2,1} &= 4\,q^4 Q_9 + \left(4\,q^3 + 2\,q^2\right) Q_{8,1} + \left(4\,q^2 + 2\,q\right) Q_{7,2} + 2\,q Q_{6,3} + Q_{6,2,1} \\
G^q_{6,3} &= 2\,q^3 Q_9 + 2\,q^2 Q_{8,1} + 2\,q Q_{7,2} + Q_{6,3} \\
G^q_{7,2} &= 2\,q^2 Q_9 + 2\,q Q_{8,1} + Q_{7,2} \\
G^q_{8,1} &= 2\,q Q_9 + Q_{8,1} \\
G^q_9 &= Q_9
\end{aligned}
$$

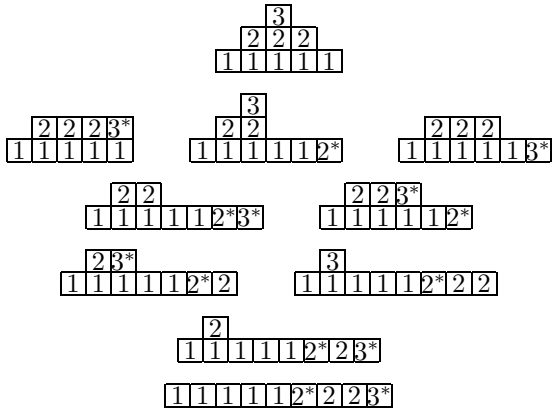
5.2. Conjectured tableaux rank function of content $(4,3,2)$.

$$
\begin{aligned}
G^q_{4,3,2} &= 4\,q^7 Q_9 + \left(4\,q^5 + 4\,q^6\right) Q_{8,1} + \left(2\,q^3 + 4\,q^5 + 8\,q^4\right) Q_{7,2} + \left(2\,q^2 + 8\,q^3 + 4\,q^4\right) Q_{6,3} \\
&\quad + \left(4\,q^3 + 2\,q^2\right) Q_{6,2,1} + \left(4\,q^3 + 2\,q^2\right) Q_{5,4} + \left(4\,q^2 + 2\,q\right) Q_{5,3,1} + Q_{4,3,2}
\end{aligned}
$$



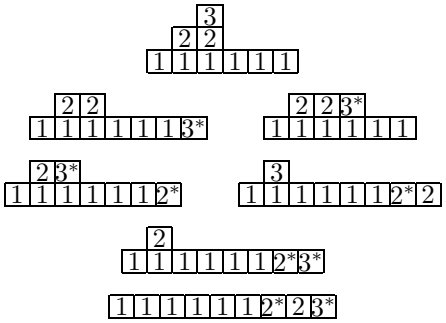
5.3. **Conjectured tableaux rank function of content** $(5,3,1)$**.**

$$G^q_{5,3,1} = 4\,q^5 Q_9 + \left(2\,q^3 + 4\,q^4\right) Q_{8,1} + \left(4\,q^3 + 4\,q^2\right) Q_{7,2} + \left(4\,q^2 + 2\,q\right) Q_{6,3} + 2\,q Q_{6,2,1} + 2\,q Q_{5,4} + Q_{5,3,1}$$



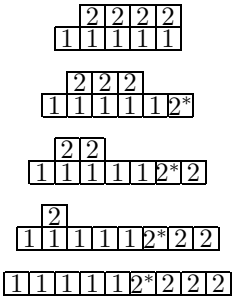
5.4. **Conjectured tableaux rank function of content** $(6,2,1)$**.**

$$G^q_{6,2,1} = 4\,q^4 Q_9 + \left(2\,q^2 + 4\,q^3\right) Q_{8,1} + \left(4\,q^2 + 2\,q\right) Q_{7,2} + 2\,q Q_{6,3} + Q_{6,2,1}$$



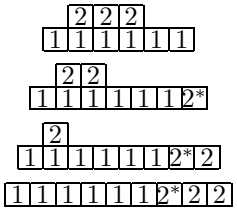
5.5. **Conjectured tableaux rank function of content** $(5,4)$**.**

$$G^q_{5,4} = 2\,q^4 Q_9 + 2\,q^3 Q_{8,1} + 2\,q^2 Q_{7,2} + 2\,q Q_{6,3} + Q_{5,4}$$



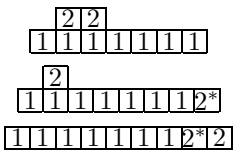
5.6. **Conjectured tableaux rank function of content** $(6,3)$**.**

$$G^q_{6,3} = 2\,q^3 Q_9 + 2\,q^2 Q_{8,1} + 2\,q Q_{7,2} + Q_{6,3}$$



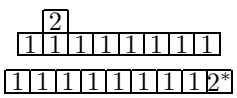
5.7. **Conjectured tableaux rank function of content** $(7,2)$**.**

$$G^q_{7,2} = 2\,q^2 Q_9 + 2\,q Q_{8,1} + Q_{7,2}$$



5.8. **Conjectured tableaux rank function of content** $(8,1)$**.**

$$G^q_{8,1} = 2\,q Q_9 + Q_{8,1}$$



6. Conjectured rank functions for $n = 10$

6.1. $G_\lambda[X;q]$ for λ strict partition of 10.

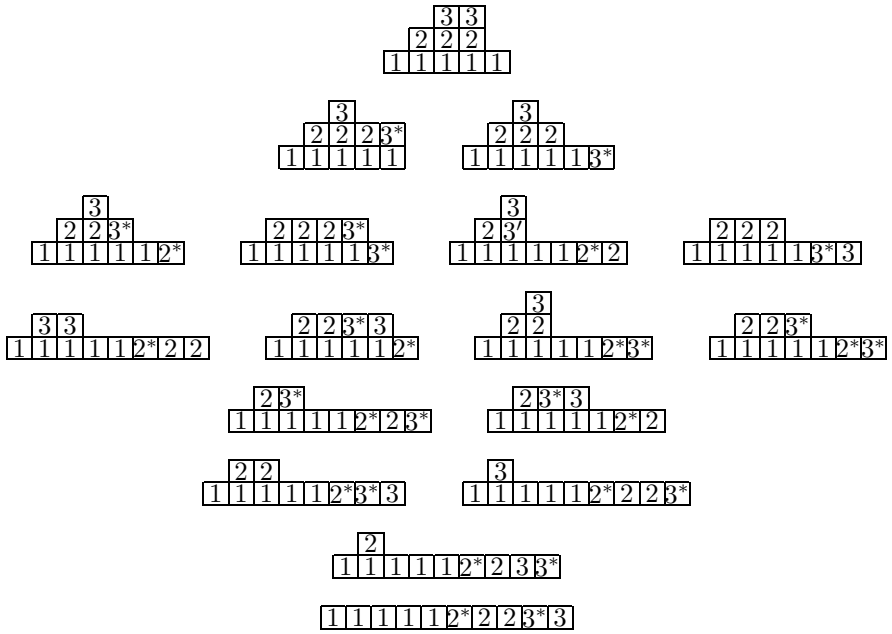
$$
\begin{aligned}
G^q_{4,3,2,1} &= 8\,q^{10}Q_{10} + \left(4\,q^7 + 8\,q^9 + 8\,q^8\right)Q_{9,1} + \left(4\,q^5 + 16\,q^7 + 16\,q^6 + 8\,q^8\right)Q_{8,2} \\
&\quad + \left(8\,q^4 + 24\,q^5 + 16\,q^6 + 8\,q^7\right)Q_{7,3} + \left(2\,q^3 + 4\,q^4 + 12\,q^5 + 8\,q^6\right)Q_{7,2,1} \\
&\quad + \left(4\,q^3 + 12\,q^4 + 8\,q^6 + 16\,q^5\right)Q_{6,4} + \left(2\,q^2 + 8\,q^3 + 20\,q^4 + 8\,q^5\right)Q_{6,3,1} \\
&\quad + \left(2\,q^2 + 8\,q^3 + 8\,q^4\right)Q_{5,4,1} + \left(2\,q + 4\,q^2 + 8\,q^3\right)Q_{5,3,2} + Q_{4,3,2,1} \\
G^q_{5,3,2} &= 4\,q^7Q_{10} + \left(4\,q^5 + 4\,q^6\right)Q_{9,1} + \left(4\,q^5 + 2\,q^3 + 8\,q^4\right)Q_{8,2} + \left(4\,q^4 + 2\,q^2 + 8\,q^3\right)Q_{7,3} \\
&\quad + \left(2\,q^2 + 4\,q^3\right)Q_{7,2,1} + \left(4\,q^2 + 4\,q^3\right)Q_{6,4} + \left(2\,q + 4\,q^2\right)Q_{6,3,1} + 2\,qQ_{5,4,1} + Q_{5,3,2} \\
G^q_{5,4,1} &= 4\,q^6Q_{10} + \left(2\,q^4 + 4\,q^5\right)Q_{9,1} + \left(4\,q^4 + 4\,q^3\right)Q_{8,2} + \left(4\,q^2 + 4\,q^3\right)Q_{7,3} \\
&\quad + 2\,q^2Q_{7,2,1} + \left(2\,q + 4\,q^2\right)Q_{6,4} + 2\,qQ_{6,3,1} + Q_{5,4,1} \\
G^q_{6,3,1} &= 4\,q^5Q_{10} + \left(2\,q^3 + 4\,q^4\right)Q_{9,1} + \left(4\,q^2 + 4\,q^3\right)Q_{8,2} + \left(2\,q + 4\,q^2\right)Q_{7,3} + 2\,qQ_{7,2,1} \\
&\quad + 2\,qQ_{6,4} + Q_{6,3,1} \\
G^q_{6,4} &= 2\,q^4Q_{10} + 2\,q^3Q_{9,1} + 2\,q^2Q_{8,2} + 2\,qQ_{7,3} + Q_{6,4} \\
G^q_{7,2,1} &= 4\,q^4Q_{10} + \left(4\,q^3 + 2\,q^2\right)Q_{9,1} + \left(4\,q^2 + 2\,q\right)Q_{8,2} + 2\,qQ_{7,3} + Q_{7,2,1} \\
G^q_{7,3} &= 2\,q^3Q_{10} + 2\,q^2Q_{9,1} + 2\,qQ_{8,2} + Q_{7,3} \\
G^q_{8,2} &= 2\,q^2Q_{10} + 2\,qQ_{9,1} + Q_{8,2} \\
G^q_{9,1} &= 2\,qQ_{10} + Q_{9,1} \\
G^q_{10} &= Q_{10}
\end{aligned}
$$

6.2. Conjectured tableaux rank function of content $(4,3,2,1)$.

$$
\begin{aligned}
G^q_{4,3,2,1} &= 8\,q^{10}Q_{10} + \left(4\,q^7 + 8\,q^9 + 8\,q^8\right)Q_{9,1} + \left(4\,q^5 + 16\,q^7 + 16\,q^6 + 8\,q^8\right)Q_{8,2} \\
&\quad + \left(8\,q^4 + 24\,q^5 + 16\,q^6 + 8\,q^7\right)Q_{7,3} + \left(2\,q^3 + 4\,q^4 + 12\,q^5 + 8\,q^6\right)Q_{7,2,1} \\
&\quad + \left(4\,q^3 + 12\,q^4 + 8\,q^6 + 16\,q^5\right)Q_{6,4} + \left(2\,q^2 + 8\,q^3 + 20\,q^4 + 8\,q^5\right)Q_{6,3,1} \\
&\quad + \left(2\,q^2 + 8\,q^3 + 8\,q^4\right)Q_{5,4,1} + \left(2\,q + 4\,q^2 + 8\,q^3\right)Q_{5,3,2} + Q_{4,3,2,1}
\end{aligned}
$$

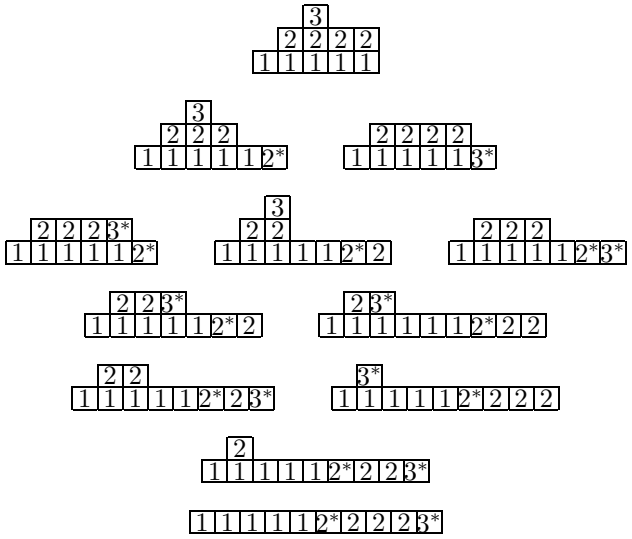
6.3. **Conjectured tableaux rank function of content** $(5,3,2)$**.**

$$G^q_{5,3,2} = 4\,q^7 Q_{10} + \left(4\,q^5 + 4\,q^6\right) Q_{9,1} + \left(4\,q^5 + 2\,q^3 + 8\,q^4\right) Q_{8,2} + \left(4\,q^4 + 2\,q^2 + 8\,q^3\right) Q_{7,3} + \left(2\,q^2 + 4\,q^3\right) Q_{7,2,1} + \left(4\,q^2 + 4\,q^3\right) Q_{6,4} + \left(2\,q + 4\,q^2\right) Q_{6,3,1} + 2\,q Q_{5,4,1} + Q_{5,3,2}$$



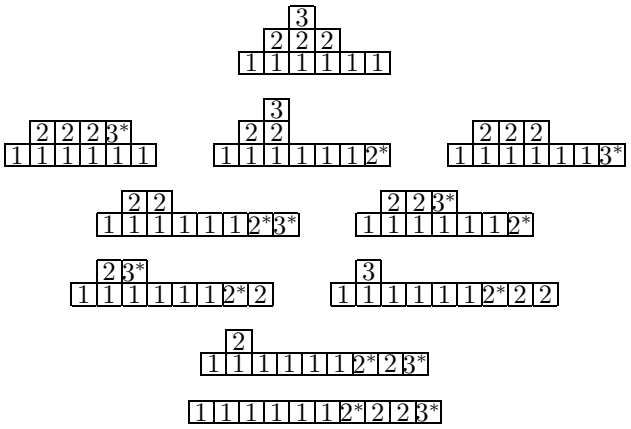
6.4. **Conjectured tableaux rank function of content** $(5,4,1)$**.**

$$G^q_{5,4,1} = 4\,q^6 Q_{10} + \left(2\,q^4 + 4\,q^5\right) Q_{9,1} + \left(4\,q^4 + 4\,q^3\right) Q_{8,2} + \left(4\,q^2 + 4\,q^3\right) Q_{7,3} + 2\,q^2 Q_{7,2,1} + \left(2\,q + 4\,q^2\right) Q_{6,4} + 2\,q Q_{6,3,1} + Q_{5,4,1}$$



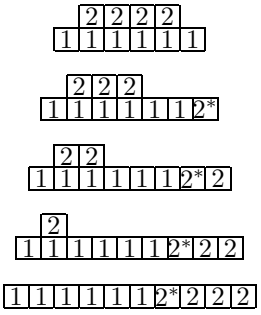
6.5. **Conjectured tableaux rank function of content** $(6,3,1)$**.**

$$G^q_{6,3,1} = 4\,q^5 Q_{10} + \left(2\,q^3 + 4\,q^4\right) Q_{9,1} + \left(4\,q^2 + 4\,q^3\right) Q_{8,2} + \left(2\,q + 4\,q^2\right) Q_{7,3} + 2\,qQ_{7,2,1} + 2\,qQ_{6,4} + Q_{6,3,1}$$



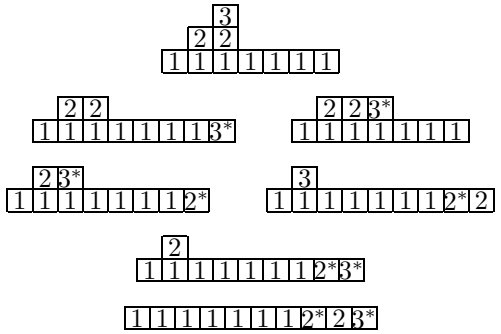
6.6. **Conjectured tableaux rank function of content** $(6,4)$**.**

$$G^q_{6,4} = 2\,q^4 Q_{10} + 2\,q^3 Q_{9,1} + 2\,q^2 Q_{8,2} + 2\,qQ_{7,3} + Q_{6,4}$$



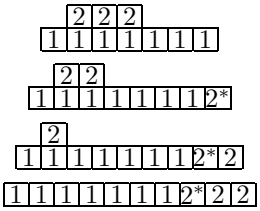
6.7. **Conjectured tableaux rank function of content** $(7,2,1)$**.**

$$G^q_{7,2,1} = 4\,q^4 Q_{10} + \left(4\,q^3 + 2\,q^2\right) Q_{9,1} + \left(4\,q^2 + 2\,q\right) Q_{8,2} + 2\,qQ_{7,3} + Q_{7,2,1}$$



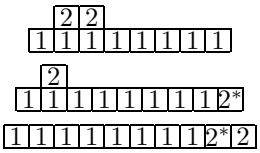
6.8. **Conjectured tableaux rank function of content** $(7,3)$**.**

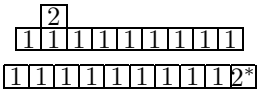
$$G^q_{7,3} = 2\,q^3 Q_{10} + 2\,q^2 Q_{9,1} + 2\,q Q_{8,2} + Q_{7,3}$$



6.9. **Conjectured tableaux rank function of content** $(8,2)$**.**

$$G^q_{8,2} = 2\,q^2 Q_{10} + 2\,q Q_{9,1} + Q_{8,2}$$



6.10. **Conjectured tableaux rank function of content** $(9,1)$**.**

$$G^q_{9,1} = 2\,q Q_{10} + Q_{9,1}$$